Does Incomplete Spanning in International Financial Markets Help to Explain Exchange Rates?

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Discussion by
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Introduction

- Paper asks if models with incomplete markets help explaining the behavior of exchange rates, and specifically
  - Low volatility of exchange rates relative to other asset prices
  - Deviations from uncovered interest rate parity
  - Low correlation between exchange rates and economic “fundamentals”

- The contribution is to develop an approach to address this question
  - Take stochastic properties of SDF as given
  - Incomplete markets modeled as a “wedge”
  - Characterize restrictions on the wedge due to trading in risk-free bonds

- Paper finds that the wedge, *per se*, cannot do much
Very useful and clean exercise. It should be thought to PhD’s students

1 Overview of the paper

2 Two main comments

• Incomplete markets and the SDF

• A more formal test?
**Complete Markets and the Exchange Rate Puzzles**

Under complete markets, and no trading restrictions across countries:

\[ M_{t+1}^* = M_{t+1} \frac{S_{t+1}}{S_t} \]

Then, we have:

- \[ \text{var}_t(\Delta s_{t+1}) = \text{var}_t(m_{t+1}^*) + \text{var}_t(m_{t+1}) - 2\text{cov}_t(m_{t+1}^*, m_{t+1}) \]

  **Volatility puzzle**: For plausible values of sdf variances, we have that \( \text{var}_t(\Delta s_{t+1}) \gg \) data, unless \( m_{t+1} \) and \( m_{t+1}^* \) highly correlated

- \[ r_t^* - \left( r_t - \Delta s_{t+1} \right) = \frac{1}{2} \left[ \text{var}_t(m_{t+1}) - \text{var}_t(m_{t+1}^*) \right] \]

  **UIP puzzle**: Hard to generate sizable deviations from UIP

- \[ \frac{\text{cov}_t(\Delta s_{t+1}, m_{t+1}^* - m_{t+1})}{\text{var}_t(\Delta s_{t+1})} = 1 \]

  **Backus-Smith puzzle**: \( \Delta s_{t+1} \) orthogonal to \( \Delta c_{t+1}^* - \Delta c_{t+1} \)
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With incomplete markets we have:

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TRADING IN RISK-FREE BONDS

Assuming that risk-free bonds are freely traded across countries, we have

$$\mathbb{E}_t[M^*_{t+1}R^*_t] = \mathbb{E}_t \left[ M_{t+1} \frac{S_{t+1}}{S_t} R^*_t \right] = 1 \quad \mathbb{E}_t[M_{t+1}R_t] = \mathbb{E}_t \left[ M^*_{t+1} \frac{S_t}{S_{t+1}} R_t \right] = 1$$

This generates restrictions on \( \{\eta_{t+1}\} \). Specifically, we must have

$$\text{cov}_t(m^*_{t+1} - m_{t+1}, \eta_{t+1}) = -\text{var}_t(\eta_t),$$

implying

- \( \text{var}_t(\Delta s_{t+1}) = \text{var}_t(m^*_{t+1} - m_{t+1}) - \text{var}_t(\eta_{t+1}) \)

- \( r^*_t - \left( r_t - \Delta s_{t+1} \right) = \frac{1}{2} \left[ \text{var}_t(m_{t+1}) - \text{var}_t(m^*_{t+1}) \right] + \mathbb{E}_t[\eta_{t+1}] \)

- \( \frac{\text{cov}_t(\Delta s_{t+1}, m^*_{t+1} - m_{t+1})}{\text{var}_t(\Delta s_{t+1})} = 1 \)
Main result: incomplete spanning might help in addressing volatility and UIP puzzle, but it does not help with Backus-Smith puzzle
Comment 1: Incomplete Markets and the SDF

- The paper takes \( \{m_{t+1}, m^*_t\} \) as given in the exercise. But market incompleteness modifies the properties of \( \{m_{t+1}, m^*_t\} \)

- In models with segmented and incomplete markets, stochastic discount factors are typically functions of the leverage of “experts”

- A growing literature documents that leverage-based SDF outperforms consumption-based SDF in explaining asset prices (Adrian, Etula and Muir, 2016; Bocola, 2016)

- Would be interesting to know if this holds true for currencies. For example, how does the Backus-Smith slope looks in the data when using a leverage-based pricing kernel?
Comment 2: A more formal test?

- Paper suggests that incomplete markets do not help much fitting the behavior of exchange rates
  - Backus-Smith puzzle
  - Need $\mathbb{E}_t[\eta_{t+1}] > 0$ to deal with UIP puzzle. But this introduces predictability in exchange rates changes

- It would be nice to have a more formal test. For example, one can use

$$\Delta s_{t+1} = m^*_{t+1} - m_{t+1} + \eta_{t+1}$$

as a measurement equation and compare marginal data densities of the incomplete market model and the model with $\eta_{t+1} = 0$

- This would give a more precise answer to the authors’ question
• Very nice paper

• Two comments/questions
  • How to think about the implications of incomplete markets for \( \{m^*_t, m_{t+1}\} \)?
  • A more formal test of the hypothesis?