Imperfect Risk Sharing and the Business Cycle∗

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Abstract

This paper studies the macroeconomic implications of imperfect risk sharing implied by a class of New Keynesian models with heterogeneous agents. The models in this class can be equivalently represented as an economy with a representative household that has state-dependent preferences. These preference wedges are functions of households’ consumption shares and relative wages, and they identify the key cross-sectional moments that are relevant for the business cycle implications of these models, making them ideal calibration targets. We measure the wedges using US household-level data, and feed them into the equivalent representative-agent economy to perform counterfactuals. We find that deviations from perfect risk sharing account for only 7% of output volatility, but can have much larger output effects when nominal interest rates reach their lower bound.

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1 Introduction

To what extent are households’ heterogeneity and deviations from perfect risk sharing important for aggregate fluctuations? Starting from the influential quasi-aggregation result in Krusell and Smith (1998), the consensus view in macroeconomics was that idiosyncratic risk and missing insurance markets were not an important driver of business cycle fluctuations. After the Great Recession, many researchers started to reassess this question. Several studies suggested that the fall in aggregate consumption was partly due to tighter debt limits and heightened households’ precautionary savings motives. These analyses, surveyed in Krueger, Mitman, and Perri (2016) and Kaplan and Violante (2018), made significant contributions by clarifying the mechanisms through which shocks and frictions at the micro level can propagate to the aggregate in economies with nominal rigidities. It remains however challenging to quantify these mechanisms, as their strength is dependent on several modeling choices, like the assumed set of risk-sharing mechanisms available to households, the nature of their idiosyncratic risk, and the timing and distribution of fiscal transfers.\footnote{A prominent example is Kaplan and Violante (2014), which shows that the consumption response to fiscal transfers is very different if households can trade one liquid asset or one liquid and one illiquid asset. Other modeling choices, which are inconsequential in representative agent economy, matter in heterogeneous agent economies such as the timing and distribution of the fiscal transfers (Kaplan, Moll, and Violante, 2018), how profits get distributed across households (Broer, Hansen, Krusell, and Oberg, 2018), and the cyclicity of idiosyncratic risk and access to liquidity (Werning, 2015).}

In this paper we propose a simple method to quantify the importance of imperfect risk sharing for the business cycle and to help researchers discipline these modeling choices. We extend a result by Nakajima (2005) and show that for a large class of economies the contribution of imperfect risk sharing to aggregate fluctuations is fully summarized by two statistics of the equilibrium cross-sectional distribution of households’ consumption shares and relative wages. We call these two statistics preference wedges, because they can be interpreted as a time-varying discount factor and disutility of labor in an otherwise standard representative agent economy. We measure these wedges using household-level data from the Consumption Expenditure Survey and use this representative-agent interpretation as a measurement device. In our application, we find that deviations from perfect risk sharing account for only 7% of output volatility, but they can have much larger effects when nominal interest rates reach their lower bound. In an event study, we find that they account for one-fourth of the output drops observed during the Great Recession.

We apply our methodology to a class of New Keynesian models with heterogeneous agents. To keep the analysis transparent, the production side of these economies is the same as in the canonical “three equations” model. Households have homogeneous isoelastic preferences, face idiosyncratic income risk, and their ability to smooth these shocks depends
on the risk-sharing mechanisms available to them, e.g. the set of financial assets they can trade, their financial constraints and the presence of a redistributive fiscal transfers. We model these features in a flexible way that nests several specifications considered in the literature including incomplete market models in the Bewley-Hugget-Aiyagari tradition as well as those with endogenous debt limits as in Kehoe and Levine (1993) and Alvarez and Jermann (2000).

Our analysis builds on an equivalence result between these economies with heterogeneous agents and the canonical representative-agent New Keynesian model. For any heterogeneous agent economy in our class, the macroeconomic aggregates—output, inflation and nominal interest rates—satisfy the equilibrium conditions of a representative-agent New Keynesian model where the stand-in household has a state-dependent discount factor and disutility of labor. We call this the representative-agent (RA) representation. The two preference wedges are functions of the joint distribution of households’ consumption shares and relative wages, so they are typically endogenous objects in the original heterogeneous agent economy. However, they identify the key cross-sectional moments that govern the macroeconomic effects of imperfect risk sharing in this class of models, in the sense that shocks and frictions at the micro level matter for the aggregate only if they generate time variation in these two statistics. Importantly, because the mapping from households’ choices to these wedges is the same for all the models in the class, they can be measured from panel data without specifying the details of the model that generates them.

The discount factor of the stand-in household in the RA representation is the product of the true rate of time-preference and the discount factor wedge. This statistic captures the failure of aggregation when consumption risk is not perfectly shared across households, and it is the expected inverse change in the consumption shares of the households that are financially unconstrained. To understand this result, consider first the case of complete financial markets. With our isoelastic preferences, the allocation features constant consumption shares, and the discount factor of the stand-in household coincides at every point in time with the true households’ rate of time preferences. When risk sharing is not perfect, consumption shares are state-dependent, and the two no longer coincide. The discount factor wedge is typically greater than 1 and can be time-varying. For example, an increase in households’ precautionary saving motives in the original heterogeneous-agents economy—due to heightened idiosyncratic risk or occasionally binding borrowing constraints—is isomorphic to an increase in the discount factor wedge in the RA representation.

In addition to a time-varying discount factor, the stand-in household also features a state-dependent preference for leisure. This wedge captures compositional changes in hours worked that occur in the original economy with heterogeneous households. For example, an increase in the cross-sectional dispersion of households’ labor productivity induces
high-productivity households to work more and low-productivity households to work less because of substitution effects in labor supply. This induces an increase in worked hours measured in efficiency units in the original heterogeneous-agents economy. In the RA representation, this compositional change in the labor force is captured by a reduction in the disutility of labor of the stand-in household—an increase in labor supply.

We use panel data from the Consumer Expenditure Survey (CEX) to measure the realization of these two wedges for the US economy over the 1992-2017 period. While the measurement of the disutility of labor is relatively straightforward, the measurement of the discount factor poses some challenges. This wedge is the expected inverse of the change in consumption share for unconstrained households. To approximate this statistic, we divide households into groups based on their income and net worth and compute cross-sectional harmonic averages of their change in consumption shares to measure the discount factor wedge for different sub-groups of the population. Under our assumptions, unconstrained households are those with the highest average discount factor wedge.\(^2\) We demonstrate that these approximations perform remarkably well in practice when using simulated data from two benchmark incomplete markets models, the Krusell and Smith (1998) and Guerrieri and Lorenzoni (2017) economies.

Having measured the wedges, we proceed to study their behavior over time. The labor disutility wedge does not display much variation at business cycle frequencies. For the discount factor wedge, we document two important facts. First, and consistent with benchmark incomplete markets models, we find that high income households have on average a higher discount factor than low income households. Through the lens of our framework, this identifies the former as the financially unconstrained agents. Second, we find that the discount factor wedge for this group increases substantially at the beginning of the Great Recession.

It is well known in the literature that an increase in the patience of the representative agent can induce sizable drops in real economic activity in the canonical New Keynesian model, especially when the zero lower bound constraint binds (Christiano, Eichenbaum, and Rebelo, 2011).\(^3\) Indeed, researchers that have estimated these models on US data typically need large increases in the discount factor to explain episodes of low interest rates, inflation and output, as for example the US Great Recession. Several recent papers have pointed out that time-varying self-insurance motives of households may be the root cause underlying

\(^2\)In our class of models, financially unconstrained households have lower average growth/volatility in consumption shares than constrained households, factors that imply a higher discount factor wedge. See also Aguiar, Bils, and Boar (2020) for a discussion of this theoretical property.

\(^3\)This is especially true for New Keynesian models that, unlike the one we study here, feature capital accumulation. Away from the zero lower bound, an increase in patience typically generates a comovement problem between consumption and investment, as first suggested by Barro and King (1984) for neoclassical models. At the zero lower bound, this does not happen because the higher discount factor can lead to higher real interest rates.
these dynamics, and the increase in the discount factor wedge we document during the Great Recession provides qualitatively some support to these views. An important question is whether the movements in the discount factor wedge that we measure using micro data are large enough to be quantitatively an important driver of the US business cycle.

To address this question, we use the realization of the wedges and data on output, inflation, and nominal interest rates to jointly estimate the structural parameters of the model and of the stochastic process governing the wedges. Given the estimated parameters, we use the RA representation to construct the counterfactual path for aggregate variables in an economy with complete financial markets—that is, an economy with time-invariant consumption shares. We show that the presence of complete financial markets reduces the standard deviation of output by only 7%, suggesting that deviations from perfect risk sharing contribute little on average to the dynamics of output over the business cycle. However, we show that these effects are more sizable during the Great Recession because of the binding zero lower bound constraint. In this episode, imperfect risk sharing accounts for roughly one-half of the increase in the discount factor that the RA New Keynesian model needs to fit the aggregate data and one-fourth of the output losses observed in 2009-2010.

We finally ask what economic mechanism can account for the increase in the measured discount factor wedge during the Great Recession. This statistic can increase because of two mutually non-exclusive reasons: the average consumption shares of unconstrained agents fell over this episode, or there was an increase in their cross-sectional dispersion. We show that this second force explains more than half of the increase in this wedge. This finding suggests that structural models aimed at capturing these patterns in the data should focus on frictions that generate uninsured idiosyncratic risk within the group of unconstrained households. A further decomposition shows that the increase in the cross-sectional dispersion of consumption shares is driven by a reduction in the ability of high-income households to smooth negative income shocks rather than by an increase in the volatility of their labor income. These findings provide support for research that emphasizes the importance of households’ credit constraints over this episode or, alternatively, to the view that income shocks occurring at the time were perceived to be particularly persistent.

Related Literature. Our research contributes to a growing literature that introduces heterogeneous agents and incomplete financial markets in New Keynesian models of the business cycle. Researchers have used these environments to study how frictions impeding risk sharing across households affect the transmission mechanism of monetary and fiscal policy. This effect is absent in models with simple form of heterogeneity, such as the “two-agent” New Keynesian model studied in Galí, López-Salido, and Vallsés (2007), Bilbié (2008), and Debortoli and Galí (2017).

See Kaplan, Moll, and Violante (2018); Auclert (2017); McKay, Nakamura, and Steinsson (2016); McKay and Reis (2016); Hagedorn, Manovskii, and Mitman (2019); Gornemann, Kuester, and Nakajima (2016); Bhandari,
and more generally the business cycle. As for the latter, the literature has stressed the interactions between households’ precautionary savings and aggregate demand: when the former increase, the latter falls, resulting in lower levels of economic activity. These changes in households’ precautionary behavior may occur via different mechanisms. Guerrieri and Lorenzoni (2017) and Jones, Midrigan, and Philippon (2018) show that a tightening of individual borrowing constraints can induce households to save more because of self-insurance. Other researchers highlight the importance of time-varying labor income risk, see for example McKay (2017), Challe, Matheron, Ragot, and Rubio-Ramirez (2017), Den Haan, Rendahl, and Riegler (2017) and Bayer, Lütticke, Pham-Dao, and Tjaden (2019) for quantitative analyses and Heathcote and Perri (2018) and Ravn and Sterk (2017) for more stylized frameworks.

All these papers consider specific departures from perfect risk sharing by imposing a given asset structure, income process, and set of borrowing constraints. We instead take a more agnostic approach about the amount of risk sharing available to households and infer it from their observed choices. These two approaches are complementary. We identify a set of cross-sectional moments that summarize all the relevant information about the macroeconomic effects of imperfect risk sharing in this class of models, and we believe these statistics offer ideal empirical targets for the calibration and evaluation of these models for that reason. However, our paper is mostly silent about the set of underlying frictions and shocks that can replicate the observed patterns of the wedges.

Applied papers in this literature often target moments of the cross-sectional distribution of marginal propensity to consume (MPCs), especially when the analysis focuses on quantifying the macroeconomic effects of fiscal transfers. Auclert, Rognlie, and Straub (2018) show that a moment of the distribution of MPCs at different time horizons —what they term intertemporal MPCs— is indeed a sufficient statistic for the response of output to fiscal shocks in a class of New Keynesian models. The preference wedges we introduce, instead, provide information on how imperfect risk sharing varies over time on average—not conditional on a specific shock.6 Our statistics are in principle easier to measure: unlike MPCs, they do not require quasi-experimental variation to be estimated consistently. However, they provide less information than MPCs about the transmission mechanism of specific shocks.

The counterfactuals that we perform are related to the business cycle accounting methodology of Chari, Kehoe, and McGrattan (2007). Beside the different focus, there are two main differences between these procedures. First, in our approach the wedges are measured using household-level observations, rather than being chosen to replicate the observed path of aggregate data. By doing so, we are able to isolate the component that are due to imperfect

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Evans, Golosov, and Sargent (2018) for recent contributions.

6We derive the relation between intertemporal MPCs and the preference wedges formally in Appendix A.
risk sharing. Second, our main quantitative experiment constructs the path for macroeconomic variables in a counterfactual economy with complete financial markets. This is not equivalent to the approach of Chari, Kehoe, and McGrattan (2007), which assesses the effects of specific wedges on the business cycle.

To best of our knowledge, the idea that aggregate variables in economies with heterogeneous agents admit an equivalent RA representation with state-dependent preferences was first developed by Nakajima (2005). In general, these time-varying wedges are endogenous equilibrium objects in the heterogeneous agent economy under consideration. Some recent papers provide a characterization of the discount factor wedge in some specific economies, and derive the mapping between this wedge and the primitives of the model. See Werning (2015) and Acharya and Dogra (2018) for example. Krueger and Lustig (2010) and Werning (2015) provide conditions under which the discount factor wedge in our representation is constant over time and it is therefore irrelevant for aggregate fluctuations. In our paper we show how these wedges can be measured using micro data and how to use the RA representation to quantify the macroeconomic implications of imperfect risk sharing.

Our approach builds on a large literature that uses data on household consumption, labor supply, and earnings to measure the degree of risk haring in the data without explicitly specifying the mechanisms through which households share risk. See for example Blundell, Pistaferri, and Preston (2008) and the survey in Jappelli and Pistaferri (2010). The paper that is closer to our approach is Heathcote, Storesletten, and Violante (2014) who use households’ optimality conditions and PSID and CEX data to measure the extent of risk sharing present in the U.S. economy. The contribution of our paper relative to this literature is to study how the measured degree of partial risk sharing affects aggregate dynamics.

Finally, there is a connection between our paper and the literature that evaluates the asset pricing implications of models with heterogeneous households. See for example Brav, Constantinides, and Geczy (2002), Vissing-Jørgensen (2002), Krueger, Lustig, and Perri (2008), and Kocherlakota and Pistaferri (2009). The goal of these papers is to estimate the stochastic discount factor with micro data given a particular form of market incompleteness. This is similar to the construction of the discount factor wedge in our approach. Clearly the scope of our analysis differs from these papers.

**Layout.** The paper is organized as follows. Section 2 introduces the class of heterogeneous agents economies at the center of our application. Section 3 derives the RA representation

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7 For example, if we were to only use aggregate data to measure the wedge in the Euler equation of the RA representation we would not be able to distinguish the discount factor wedge due to imperfect risk sharing from any other friction that would take the form of an Euler equation wedge. By using micro-data, we are able to separately identify the former.
and discusses why this representation is a useful tool for the evaluation of heterogeneous agents models. Section 4 discusses how we estimate the preference wedges using panel data and approximate their stochastic process. In Section 5 we use the CEX to measure the preference wedges and combine these series with the RA representation to measure the role of imperfect risk sharing for the US business cycle. Section 6 discusses possible models that are consistent with the pattern we identify in the micro data. Section 7 concludes.

2 New Keynesian models with heterogeneous agents

In this section we introduce a class of New Keynesian models with isoelastic preferences and idiosyncratic income risk. The models in this class share the same specification for households’ preferences, technology, market structure and the conduct of monetary policy—elements that are borrowed from the prototypical “three equations” New Keynesian framework. However, these models can differ in the cyclicity of idiosyncratic risk faced by households, the set of assets they can trade, their financial constraints, as well as the timing and distribution of fiscal transfers.

Time is discrete and indexed by \( t = 0, 1, \ldots \). The economy is populated by a continuum of households, final good producers, intermediate good firms, and the monetary authority. There are two types of states: aggregate and idiosyncratic. We denote the aggregate state by \( z_t \) and the idiosyncratic state by \( v_t \), both of which are potentially vector valued. Let \( z^t = (z_0, z_1, \ldots, z_t) \) be a history of realized aggregate states up to period \( t \) and \( v^t = (v_0, v_1, \ldots, v_t) \) be a history of idiosyncratic states up to period \( t \). We also let \( s_t = (z_t, v_t) \) and \( s^t = (z^t, v^t) \).

Let \( \Pr(s^t | s^{t-1}) \) be the probability of a history \( s^t \) given \( s^{t-1} \). We assume that \( \Pr(s^t | s^{t-1}) = \Pr(v^t | v^{t-1}, z^t) \Pr(z^t | z^{t-1}) \) and allow for the possibility that the aggregate states affect the distribution of the idiosyncratic states.

Households are infinitely lived and have preferences over consumption, \( c(s^t) \), and hours worked, \( l(s^t) \), given by

\[
\sum_{t=0}^{\infty} \beta^t \Pr(s^t | s_0) \tilde{\theta}(z^t) U(c(s^t), l(s^t)),
\]

where \( \beta \) is the discount factor and \( \tilde{\theta}(z^t) \) is a shock to the marginal utility of consumption and disutility of labor defined recursively as \( \tilde{\theta}(z^{t+1}) = \theta(z_{t+1})\tilde{\theta}(z^t) \). This preference shock is commonly used in the literature to obtain a binding zero lower bound constraint, a feature that will be important in our quantitative analysis. We further assume that the period utility
is given by
\[ U(c, l) = \frac{c^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{l^{1+\psi}}{1+\psi}, \tag{2} \]
with \(\sigma > 0\) and \(\psi > 0\).

The final good is produced combining differentiated intermediate goods according to the technology
\[ Y(z^t) = \left( \int_0^1 y_j(z^t)^{\frac{1}{\mu}} dj \right)^\mu, \tag{3} \]
where \(\mu\) is related to the (constant) elasticity of substitution across varieties, \(\varepsilon\), by the following, \(\mu = \varepsilon / (\varepsilon - 1)\). The intermediate inputs indexed by \(j\) are produced using labor
\[ y_j(z^t) = A(z_t)n_j(z^t), \tag{4} \]
where \(A(z_t)\) is an aggregate technology shock, common across firms, and \(n_j(z^t)\) is labor in efficiency units utilized by the producer of intermediate good \(j\). Feasibility requires that
\[ \int n_j(z^t) dj = \sum_{v^t} \Pr(v^t|z^t) e(v_t) l(v^t, z^t), \tag{5} \]
where the right hand side is the supply of labor in efficiency units by households. Each individual \(v^t\) is associated to a particular level of efficiency \(e(v_t)\): hiring more high-efficiency types, holding total hours worked fixed, results in higher output produced by the firm. This individual-specific productivity shock \(e(v_t)\) generates idiosyncratic income risk for households.

We now describe the market structure for this economy with a particular emphasis on the households side.

**Households.** Households enter the period with financial assets and they work for intermediate good producers. They choose consumption, new financial positions and labor in order to maximize their expected life-time utility.

We model financial markets in a flexible way. Households can trade a risk-free nominal bond. We denote by \(b(s^t)\) the position taken today by a household and by \(1 + i (z^t)\) the nominal return on the bond. Households can also trade a set \(K\) of additional assets, with the nominal payout of a generic asset \(k \in K\) given by \(R_k(s^t, s_{t+1})\). We let \(q_k(z^t, v^t)\) be the price of the asset. This formulation allows for different types of financial assets: individual Arrow securities, shares of the intermediate good firms, complex financial derivatives, etc. We let \(a_k(z^{t-1}, v^{t-1})\) be the holdings of assets \(k\) that a household with history \(v^{t-1}\) has accumulated after an aggregate history \(z^{t-1}\). Trades in these additional financial assets potentially require
transaction costs $T(\{a_k(s^{t-1})\}_{k \in K}, \{a_k(s^t)\}_{k \in K}, s^t)$ that can depend on the inherited portfolio \(\{a_k(s^{t-1})\}_{k \in K}\), the new portfolio \(\{a_k(s^t)\}_{k \in K}\), and \(s^t\).

In addition, we allow for a number of constraints that potentially restricts the financial positions that households can choose,

$$ H \left(b \left( s^t \right), \{a_k \left( s^t \right)\}_{k \in K}, s^t\right) \geq 0 $$

for some vector-valued function $H$. We refer to the set of constraints in (6) as trading restrictions.

The set of assets $K$, the transaction costs $T$, and the trading restrictions in (6) are a flexible way of representing different sets of risk-sharing mechanisms available to households. The only restriction that we impose is that purchasing risk-free nominal bonds weakly relaxes these constraints, $\partial H \left(b, \{a_k\}_{k \in K}, s^t\right) / \partial b \geq 0$, and does not require a transaction cost. By doing so we are ruling out limited participation economies where agents must pay a fixed cost to have access to the risk-free nominal bond. Our formulation nests the complete financial market case, when the set of tradable assets spans all possible aggregate and idiosyncratic histories and there are no transaction costs or trading restrictions. In addition, it encompasses as special cases a large class of models with incomplete financial markets: the Bewley-Huggett-Aiyagari economy, the two-assets economy in Kaplan and Violante (2014) and Kaplan, Moll, and Violante (2018), the endogenous debt limits in Alvarez and Jermann (2000), or the various restrictions on asset trading in Chien, Cole, and Lustig (2011, 2012). Note, also, that the $H$ function can depend on $s^t$, which implies that we are allowing for aggregate and idiosyncratic shocks to affect the financial constraints of households. Moreover, our formulation allows for heterogeneity in households access to assets other than the risk-free nominal bond, a property that is critical to account for the wealth distribution in the data.

Given initial assets’ holdings, households choose \(\{c(s^t), l(s^t), b(s^t), \{a_k(s^t)\}_{k \in K}\}\) to maximize utility (1) subject to the trading restrictions in (6) and the nominal budget constraint,

$$ P \left(z^t\right) c \left(s^t\right) + \frac{b \left(s^t\right)}{1 + i \left(z^t\right)} + \sum_{k \in K} q_k(s^t)a_k \left(s^t\right) + T \left(\{a_k(s^{t-1})\}_{k \in K}, \{a_k(s^t)\}_{k \in K}, s^t\right) $$

$$ \leq W \left(z^t\right) e(v_0) l \left(v^t, z^t\right) - T(s^t) + b \left(s^{t-1}\right) + \sum_{k \in K} R_k \left(s^{t-1}, s_t\right) a_k \left(s^{t-1}\right), $$

where $W \left(z^t\right)$ is the nominal wage per efficiency units and $T(s^t)$ are lump-sum taxes. To ease on notation, initial asset holdings are indexed by the initial idiosyncratic state $v_0$. 


Because of the assumption that $\frac{\partial H}{\partial b} \geq 0$, a necessary condition for optimality is

$$\frac{1}{1 + i(z^t)} \geq \beta \sum_{s_{t+1}} \left\{ \frac{\Pr(s_{t+1}|s^t) \theta(z_{t+1})}{1 + \pi(z_{t+1})} \left[ \frac{c(s^t, s_{t+1})}{c(s^t)} \right]^{-\sigma} \right\},$$

(7)

where $\pi(z_{t+1}) = P(z_{t+1})/P(z^t) - 1$ is the net inflation rate. The condition must hold with equality if the trading restrictions on the nominal bond do not bind. For the rest of the paper, we assume that there always exist an agent for which equation (7) holds as an equality.\(^8\) Because we have assumed that $\frac{\partial H}{\partial b} \geq 0$, equation (7) holds with equality for the agents with the highest valuation for the risk-free bond.\(^9\) Moreover, labor supply must satisfy

$$c(s^t)^{-\sigma} w(z^t) e(v_t) = \chi l(s^t)^{\phi}$$

(8)

where $w(z^t) = W(z^t)/P(z^t)$ is the real wage per efficiency unit.

**Final good producers.** The final good is produced by competitive firms that operate the production function in (3). From their decision problem, we can derive the demand function for the $j$-th variety

$$y_j(z^t) = \left( \frac{P_j(z^t)}{P(z^t)} \right)^{\mu/(1-\mu)} Y(z^t)$$

(9)

where $P_j(z^t)$ is the price of variety $j$ and $P(z^t) = \left[ \int P_j(z^t)^{1/(1-\mu)} \, dj \right]^{1-\mu}$ is the price index.

**Intermediate good producers.** Each intermediate good is supplied by a monopolistic competitive firm. The monopolist of variety $j$ operates the technology (4). The firm faces quadratic costs to adjust their prices relative to the economy’s inflation target,

$$\frac{\kappa}{2} \left[ \frac{P_j(z^t)}{P_j(z^t-1)(1+\pi)} - 1 \right]^2,$$

(10)

where $\pi \hat{=} \pi$ is the inflation target of the monetary authority.

The problem of firm $j$ is to choose its price $P_j(z^t)$ given its previous price $P_j(z^t-1)$ to maximize the present discounted value of real profits. We assume that the firm discounts

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\(^8\)This assumption implicitly imposes restrictions on the set of additional assets available, the trading restrictions, and the shocks. As an example, the assumption is automatically satisfied in a Huggett economy where the only asset available is the nominal risk-free bond in zero net supply and the only trading restriction is a debt limit of the form $b \geq -\phi$ with $\phi > 0$.

\(^9\)To see this, simply note that the agents that attains the maximum in the right side of (7) are the ones with the lowest multipliers on the trading restriction constraints.
future profits using the real state price

\[
Q(z^{t+1}) = \beta \max_{v^t} \left\{ \Pr(z^{t+1}|z^t) \theta(z_{t+1}) \sum_{v_{t+1}} \Pr(v^{t+1}|z^{t+1}, v^t) \left[ \frac{c(z^{t+1}, v^{t+1})}{c(z^t, v^t)} \right]^{-\sigma} \right\}. \tag{11}
\]

That is, firms discount future profits using the marginal rate of substitution of the agent that values dividends in the aggregate state the most.\(^\text{10}\) The firm’s problem can be written recursively as

\[
V(P_j, z^t) = \max_{p_j, y_j, n_j} \frac{P_j y_j}{P(z^t)} - w(z^t)n_j(z^t) - \kappa \left[ \frac{P_j}{P_j(1 + \pi)} - 1 \right]^2 + \sum_{z^{t+1}} Q(z^{t+1}|z^t)V(P_j, z^{t+1}) \tag{12}
\]

subject to the production function (4) and the demand function (9).

The solution to the firm’s problem together with symmetry across firms requires that the following version of the New Keynesian Phillips curve holds in equilibrium

\[
\bar{\pi}(z^t) = \frac{1}{\kappa(\mu - 1)} Y(z^t) \left[ \frac{\bar{w}(z^t)}{A(z^t)} - 1 \right] + \sum_{z^{t+1}} Q(z^{t+1}|z^t) \bar{\pi}(z^{t+1}) \tag{13}
\]

where we define \(\bar{\pi}(z^t) = \left( (\pi(z^t) - \bar{\pi}) / (1 + \bar{\pi}) \right) \times \left( (\pi(z^t) + 1) / (1 + \bar{\pi}) \right)\) and \(w(z^t) / A(z^t)\) is the real marginal cost for producing a unit of the final good.

**Monetary policy and market clearing.** We assume that the monetary authority follows a standard Taylor rule

\[
1 + i(z^t) = \max \left\{ |1 + i(z^{t-1})|^{\rho_i} \left[ 1 + \tilde{i} \left( \frac{1 + \pi(z^t)}{1 + \bar{\pi}} \right)^{\gamma_p} \left( \frac{Y(z^t)}{Y^{\text{pot}}(z^t)} \right)^{\gamma_y} \right]^{1 - \rho_i} \exp\{\epsilon_m(z_t)\}, 1 \right\}, \tag{14}
\]

where \((1 + \tilde{i}) = (1 + \pi) / \beta\) is the nominal interest in a deterministic steady state of the model, \(Y^{\text{pot}}(z^t)\) is the level of output that would prevail in an economy with no price-adjustment costs, and \(\epsilon_m(z_t)\) is a monetary shock. Because of the zero lower bound, whenever the interest rate predicted by the Taylor rule is negative, the monetary authority sets the nominal interest rate to zero.

The evolution of the aggregate supply of the nominal bond, \(B(z^t)\), and taxes, \(T(s^t)\), must satisfy the government budget constraint,

\[
B(z^{t-1}) = \frac{B(z^t)}{1 + i(z^t)} + \sum_{v^t} \Pr(v^t|z^t) T(z^t, v^t). \tag{15}
\]

\(^{10}\)If all agents could trade Arrow securities contingent on the aggregate state then this would be the equilibrium state price.
In equilibrium, the labor market, goods markets, and financial markets clear. Specifically, market clearing in the nominal bond market requires that
\[
\sum_{v^t} \Pr(v^t|z^t) b(z^t, v^t) = B(z^t). \tag{16}
\]
Since firms’ equity is the only asset in positive net supply other than the nominal risk-free bond, market clearing in all the other assets requires that the value of inherited assets must equal the nominal value of the firm cum-dividend,
\[
\sum_{v^t} \Pr(v^t|z^t) \sum_{k \in K} R_k(s^t-1, s_t) a_k(s^t-1) = P(z^t) V(P(z^t-1), z^t), \tag{17}
\]
and the total value of new asset positions equal the nominal value of the firm ex-dividend,
\[
\sum_{v^t} \Pr(v^t|z^t) \sum_{k \in K} q_k(s^t) a_k(s^t) = P(z^t) \sum_{z^t+1} Q(z^t+1|z^t) V\left(P(z^t), z^t+1\right). \tag{18}
\]
We can then define an equilibrium for this economy.

**Definition 1.** Given an asset structure \((K, T, R_k, H)\), the distribution of initial assets and lagged prices, an equilibrium is a set of households’ allocations \(\{c(s^t), l(s^t), b(s^t), a_k(s^t)\}\), a fiscal policy \(\{B(z^t), T(s^t)\}\), prices \(\{P(z^t), W(z^t), P_j(z^t), Q(z^t), q_k(z^t)\}\), and aggregates \(\{C(z^t), Y(z^t)\}\) such that i) the households’ allocation solves the households’ decision problem, ii) the price for the final good solve (12) with \(P(z^t) = P_j(z^t)\), iii) the state price is given by (11), iv) the nominal interest rate satisfies the Taylor rule (14), v) the government budget constraint (15) is satisfied, and vi) markets clear in that (16)–(18) hold and
\[
Y(z^t) = C(z^t) + \frac{\kappa}{2} \left[\frac{\pi(z^t) - \bar{\pi}}{1 + \bar{\pi}}\right]^2 + T(z^t)
\]
where aggregates are given by
\[
Y(z^t) = A(z_t) \sum_{v^t} \Pr(v^t|z^t) e(v^t) l(v^t, z^t),
\]
\[
C(z^t) = \sum_{v^t} \Pr(v^t|z^t) c(z^t, v^t),
\]
and \(T(z^t)\) are the aggregate transaction costs,
\[
T(z^t) = \sum_{v^t} \Pr(v^t|z^t) T(\{a_k(s^t-1)\}_{k \in K}, \{a_k(s^t)\}_{k \in K}, s^t).
Before moving forward, it is useful to note that while we have been flexible on the households’ block of the model, we made restrictive assumptions about other aspects of the model. For instance, there is no capital accumulation in this economy, wages are perfectly flexible, all movements in labor are at the intensive margin, and we have taken a stand on some of the aggregate shocks driving the economy—the technology, monetary and preference shock. This is a deliberate choice because it keeps our analysis transparent, within the context of the canonical “three-equations” New Keynesian model. It is fairly straightforward to conduct our analysis under a different set of assumptions regarding technology, the aggregate structural shocks, and market structure.\footnote{For example, in the Monte Carlo analysis of Section 4.3 we consider an economy that admits movements in hours worked at the extensive margin. In Appendix B we study an economy with flexible prices and capital accumulation.}

3 The RA representation

We now show that the aggregate variables in this class of New Keynesian models can be equivalently derived from the equilibrium conditions of a representative agent economy where the stand-in household has a time-varying rate of time preference and a time-varying disutility of labor. We will refer to this as the \textit{RA representation}. Section 3.1 derives this representation, while Section 3.2 provides some intuition about what economic mechanisms these time-varying preference parameters capture by deriving the RA representation for some special cases of our framework. In Section 3.3 we discuss how to use this representation to evaluate models with heterogeneous agents and to measure the macroeconomic effects of imperfect risk sharing.

3.1 Equilibrium representation

Toward establishing the RA representation for our class of New Keynesian models, let us define the following statistics

\begin{equation}
\beta(v^t, z^{t+1}) \equiv \sum_{v_{t+1}} \Pr(v_{t+1}|v^t, z^{t+1}) \left( \frac{c(z^{t+1}, v^t, v_{t+1})/C(z^{t+1})}{c(z^t, v^t)/C(z^t)} \right)^{-\varphi} \quad (19)
\end{equation}

\begin{equation}
\omega(z^t) \equiv \left[ \sum_{v^t} \Pr(v^t|z^t) \left( \frac{c(z^t, v^t)}{C(z^t)} \right)^{\frac{\varphi}{\psi}} e^{(v^t)^{1+\psi}} \right]^{-\psi}. \quad (20)
\end{equation}

We then have the following proposition where we assume that the aggregate transaction costs are negligible, $\mathcal{T}(z^t)$.
Proposition 1. Suppose that \{C(z^t), Y(z^t), \pi(z^t), i(z^t), Q(z^{t+1})\} are part of an equilibrium of an heterogeneous agent economy described in Section 2. Then, they must satisfy the aggregate Euler equation,

\[
\frac{1}{1 + i(z^t)} = \beta \max_{v^t} \sum_{z_{t+1}} \left\{ \frac{Pr(z_{t+1}^t | z^t) \theta(z_{t+1}) \theta(v^t, z_{t+1})}{1 + \pi(z_{t+1})} \left( \frac{C(z_{t+1})}{C(z^t)} \right)^{-\sigma} \right\}, \tag{21}
\]

the Phillips curve,

\[
\tilde{\pi}(z^t) = \frac{Y(z^t)}{\kappa (\mu - 1)} \left[ \mu Y(z^t)^{\psi} C(z^t)^{\sigma} \omega(z^t) \right] - 1 + \sum_{z_{t+1}} Q(z_{t+1}^t | z^t) \tilde{\pi}(z_{t+1}^t) \tag{22}
\]

the Taylor rule (14), the resource constraint

\[
Y(z^t) = C(z^t) + \frac{\kappa}{2} \left[ \frac{\pi(z^t) - \tilde{\pi}}{1 + \pi} \right]^2, \tag{23}
\]

and

\[
Q(z_{t+1}^t | z^t) = \beta \max_{v^t} \left\{ \beta(v^t, z_{t+1}) \Pr(z_{t+1}^t | z^t) \theta(z_{t+1}) \left( \frac{C(z_{t+1})}{C(z^t)} \right)^{\sigma} \right\}, \tag{24}
\]

given \{\beta(v^t, z_{t+1}), \omega(z^t)\} defined in (19) and (20).

The proof for this result is straightforward, and it extends the result of Nakajima (2005) to an economy with nominal rigidities. The aggregate Euler equation (21) is obtained by using (19) to substitute for the marginal rate of substitution \((c(s^{t+1})/c(s^t))^{-\sigma}\) in the individual Euler equation (7) and noting that under our assumptions it holds with equality for the agent with the highest marginal valuation of the bond— the "max" in equation (21). The Phillips curve (22) can be derived by substituting for the wage in (13) using the individual labor supply decisions. Indeed, multiplying both side of equation (8) by \(e(v_i)/C(z^t)^{-\sigma/\psi}\) and averaging both sides across individuals we obtain

\[
w(z^t)^{\frac{1}{\psi}} \left[ \sum_{v^t} \Pr(v^t | z^t) \left( \frac{C(z^t, v^t)}{C(z^t)} \right)^{\frac{\psi}{1+\psi}} e(v_i)^{\frac{1+\psi}{\psi}} \right] = \chi^{\frac{1}{\psi}} \left[ \sum_{v^t} \Pr(v^t | z^t) e(v_i) I(s^t) \right] C(z^t)^{\frac{\psi}{\psi}}. \tag{25}
\]

We can then use the production function (4) to express the real wage as

\[
w(z^t) = \chi \left[ \frac{Y(z^t)}{A(z_t)} \right]^{\psi} C(z^t)^{\sigma} \omega(z^t), \tag{26}
\]

and substitute it in equation (13) to obtain the Phillips curve (22). To obtain (23) we simply
substitute the adjustment costs (10) in the resource constraint.

We can easily see that equations (14), (21), (22), (23) and (24) are equivalent to those of a representative agent economy where the stand-in household has a time-varying rate of time preference given by $\beta(v^t, z^{t+1})$, and a time-varying disutility of labor given by $\omega(z^t)$. Importantly, these time-varying preferences are not structural shocks, because they are functions of the allocation in the original heterogeneous agent economy. So, they are related to the concept of a wedge in Chari, Kehoe, and McGrattan (2007). We will refer to $\{\beta(v^t, z^{t+1}), \omega(z^t)\}$ as preference wedges (or just wedges).

The representation is useful because it clarifies how households’ idiosyncratic risk and incomplete financial markets affect the behavior of output, inflation and nominal interest rates in New Keynesian models with incomplete financial markets. Shocks and frictions at the micro-level manifest themselves as shocks to the discount factor or to the disutility of working in an otherwise standard representative-agent economy. The aggregate implications of these disturbances in the New Keynesian model are well understood: a positive discount factor shock pushes down inflation and output, especially when the monetary authority is constrained by the zero lower bound, while an increase in the disutility of labor leads to an increase in inflation and nominal interest rates, and a reduction in output.

It is important to note from the aggregate Euler equation (21) that a high realization of $\beta(v^{t-1}, z^t)$ matters for output and other aggregates in period $t$ only to the extent that it predicts a higher expected discount factor between $t$ and $t+1$, $\sum_{t+1} \Pr(z^{t+1}|z^t) \beta(v^t, z^{t+1})$. For instance, if $\beta(v^{t-1}, z^t)$ is iid over time, then a high realization in period $t$ does not forecast higher future discount factor and, therefore, it has no impact on aggregates. Instead, the realization of $\omega(z^t)$ has contemporaneous effects via the Phillips curve (22).

### 3.2 Some examples

We now illustrate Proposition 1 using two examples. We start by considering the complete markets benchmark. We then study an economy where households’ idiosyncratic income risk is time-varying. This latter example shows that an increase in idiosyncratic labor income risk and the consequential increase in households’ precautionary saving motives can be equivalently represented as an increase in the discount factor of the stand-in household in the RA representation. This mechanism linking precautionary savings and aggregate demand has been studied recently by Ravn and Sterk (2017) and Heathcote and Perri (2018) among others. In Appendix A we provide additional examples, one of which based on the model of Eggertsson and Krugman (2012). There, we show that a tightening of households’ credit constraints in the original heterogeneous agent economy also maps into an increase in the discount factor in the RA representation.
These two examples illustrate the two forces that can generate fluctuations in $\beta(v^t, z^{t+1})$. As shown in Krueger and Lustig (2010), if there is no time-variation in idiosyncratic labor income risk or in the households’ ability to smooth income shocks (e.g. borrowing constraints), then $\beta(v^t, z^{t+1})$ is constant over time and the heterogeneous agent economy is equivalent to a representative agent economy with a potentially different, but time-invariant, discount factor.

Readers less interested in these examples can move to Section 3.3 without losing the thread.

Example 1: Complete markets. Suppose the set of assets $\mathcal{K}$ contains Arrow securities contingent on the realizations of the aggregate and idiosyncratic state and there are no trading restrictions other than a non-binding no-Ponzi condition. The equilibrium outcome in this economy is Pareto efficient and both aggregate and idiosyncratic risk are shared efficiently. Given our isoelastic preferences in (2), this implies that individual consumption is a constant fraction of aggregate consumption,

$$c(z^t, v^t) = \varphi(v_0) C(z^t)$$

for some consumption share $\varphi(v_0)$ constant over time and histories. Thus the discount factor in (19) for the stand-in household in the equivalent representation equals 1 for all histories, $\beta^{cm}(v^t, z^{t+1}) = 1$. The disutility of labor for the stand-in household in (20) is given by

$$\omega^{cm}(z^t) = \left[ \sum_{v^t} \text{Pr}(v^t|z^t) \varphi(v_0) \frac{v^t}{\nu} e(v^t) \frac{1 + \psi}{\psi} \right]^{-\psi}.$$ (25)

Note that the above expression can be time-varying, which implies that the complete market economy will typically differ from the representative agent economy up to $\omega^{cm}(z^t)$. To understand this expression, suppose that $\psi = 1$ and households have the same initial wealth, so that $\varphi(v_0) = 1$. In this case, $\omega(z^t)$ equals the inverse of the cross-sectional variance in households’ idiosyncratic productivity: a higher cross-sectional variance of households’ productivity is captured by a reduction in the disutility of labor in the RA representation. This decline in $\omega(z^t)$ captures a change in the composition of the labor force that occurs in the original economy with heterogeneous agents.\textsuperscript{12}

\textsuperscript{12}When the cross-sectional variance of $e(v^t)$ increases, the labor supplied by high productivity households increases and the one supplied by the low productivity households decline because of a substitution effect. So, labor supply goes up when measured in efficiency units. This effect is equivalently captures by a decline in $\omega(z^t)$ in the RA representation—an increase in the labor supply of the stand-in household.
Example 2: Labor income risk and aggregate demand. Let \( \sigma = 1 \) and the idiosyncratic productivity shocks evolve according to

\[
\Delta \log[e(v_t)] = -\frac{\sigma^2(z_t)}{2} + \epsilon_t \quad \epsilon_t | z^t \sim \mathcal{N} \left( 0, \sigma^2(z_t) \right).
\]

That is, idiosyncratic productivity is a random walk with Gaussian shocks. The standard deviation of individual productivity growth varies over time with the aggregate state \( z^t \): when \( \sigma^2(z_t) \) is high, households face higher idiosyncratic risk.

To obtain analytical expressions for the \( \beta(v_t, z_t+1) \) and the \( \omega(z^t) \) implied by this model, we assume that households can only trade the risk-free bond and face the borrowing limit \( b(s_t) \geq 0 \). Because households cannot trade stocks of the firms, we also assume that the government levies taxes on the intermediate good producers and transfers the profits to the households in proportion to the realization of idiosyncratic productivity, \( e(v_t)T(z_t) \).

The tight borrowing limits, coupled with the fact that bonds are in zero net-supply, implies that households in equilibrium cannot save. Thus, all households are hand-to-mouth and every period every period consume their cash on hand, \( e(v_t) [w(z_t)l(s_t) + T(z_t)] \). Furthermore, we can verify from the labor supply condition (8) and \( \sigma = 1 \) that \( l(s_t) \) is the same across individuals. So, we have \( c(s_t) = e(v_t)C(z_t) \) from the aggregate resource constraint.

Given the equilibrium consumption function, the relative marginal rate of substitution of the households are just functions of the idiosyncratic income process,

\[
\frac{e(v_t)C(z_t)}{e(v_t)C(z_t+1)} = \frac{e(v_t)}{e(v_{t+1})}.
\]

Substituting these expressions in equation (19) and (20) we can compute the implied \( \beta(v_t, z_t+1) \) and \( \omega(z^t) \) in this specific model:

\[
\beta(v_t, z_t+1) = \sum_{v_t} \Pr(v_{t+1} | v_t, z_t) \exp \{-\Delta \log[e(v_{t+1})]\}
= \exp\{\sigma^2(z_{t+1})\}
\]

and \( \omega(z_t) = 1 \). These two expressions, coupled with equations (14), (21), (22) and (23), are enough to characterize the law of motion for aggregate variables in this specific example.

This example is useful to understand how the interaction between idiosyncratic risk and incomplete financial markets can affect aggregate variables in this class of models. Suppose that households face today higher idiosyncratic risk, that is they expect higher \( \sigma^2(z_{t+1}) \). If

\[13\text{The literature refers to this example with tight borrowing limits and bonds in zero net supply as the zero liquidity limit. See Werning (2015) and Ravn and Sterk (2017) for example.}\]
financial markets were complete, this shocks would not have any effects on the allocation. Because of incomplete financial markets, however, households have a precautionary motive to save in the risk-free bond. This increase in the propensity to save at the micro level can be represented as an increase in $\sum_{z_t} \Pr(z_{t+1}|z_t)\beta(v^t, z_{t+1})$.

3.3 Using the RA representation for model evaluation

Proposition 1 has two main implications. The first implication is that $\{\beta(v^t, z_{t+1}), \omega(z^t)\}$ summarize all the information from the “micro block” of the model that is needed to characterize the behavior of aggregate variables. That is, they are sufficient statistics for the specifics of the model regarding the set of assets traded, the transaction costs and trading restrictions faced by households, the fiscal policy $\{B(z^t), T(s^t)\}$, and the nature of their idiosyncratic income risk. The second implication is that the mapping between individual allocations and these wedges is invariant to the details of the micro block, in the sense that for all the economies nested in the environment of Section 2, the relation between $\{\beta(v^t, z_{t+1}), \omega(z^t)\}$ and the households’ allocation is the same and it is given by equation (19) and (20).

These two properties, in turn, make the RA representation a useful device for evaluating detailed economies that are nested in the class of models of Section 2. Specifically, we see two main applications of such framework.

First, suppose that we have a procedure to measure $\{\beta(v^t, z_{t+1}), \omega(z^t)\}$ using households’ level observations on consumption shares and labor productivities. Because these are sufficient statistics for how the “micro block” affects macroeconomic variables, they also provide an informative empirical target for model evaluation. That is, a researcher that wishes to use a specific model in this class for the analysis of aggregate fluctuations should make sure that his/her model reproduces the patterns in $\{\beta(v^t, z_{t+1}), \omega(z^t)\}$ that we observe in the data. This offers a minimal, yet important, test for discriminating among the different models nested in our framework.

Second, and because of the invariance property discussed above, we can use the RA representation to measure the macroeconomic implications of imperfect risk sharing without taking a stand on the details of the micro block. To understand this point, let us assume for the moment that we know the probability distribution of $z^t$ and the stochastic process for $\{\theta(z^t), A(z^t), \varepsilon_m(z^t), \beta(v^t, z_{t+1}), \omega(z^t)\}$. Thus, given a realization of $z^t$, we can use the RA representation in Proposition 1 to obtain the underlying equilibrium path for aggregate variables—output, inflation and nominal interest rates. In order to assess the macroeconomic implications of imperfect risk sharing, we want to compare these actual paths to those that would arise in an economy where idiosyncratic risk is insurable—that is, the counterfactual paths in an economy with complete financial markets. We refer to these as the complete
In Section 3.2 we have seen that the economy with complete financial markets implies that $\beta(v_{t}, z_{t+1}) = 1$ and $\omega(z_{t}) = \omega^{cm}(z_{t})$ given by (27). Because of this difference, it also features a different behavior for the aggregate variables. The comparison between the actual and the complete markets paths isolates the impact that imperfect risk sharing has for macroeconomic aggregates over the particular history $z_{t}$.

In what follows, we will show how to perform this counterfactual in an application to the Great Recession in the US. Several researchers have suggested that the deep decline in real economic activity during the Great Recession was partly induced by an increase in households’ propensity to save, either because of an increase in precautionary motives or because of a tightening of individual’s borrowing constraints. If these mechanisms were important, we should observe the actual output trajectory to be substantially below its complete market counterfactual when feeding the history $z_{t}$ that led to the Great Recession.

In order to carry out this exercise, we need a procedure to measure the realization of $\{\beta(v_{t}, z_{t+1}), \omega(z_{t})\}$ using households’ level data and to approximate their stochastic process. We now turn to discuss these two issues.

# 4 Measuring the wedges

In Section 4.1 we propose an approach to estimate the realization of $\{\beta(v_{t}, z_{t+1}), \omega(z_{t})\}$ using panel data. We denote such realization by $\{\beta_{i,t+1}, \omega_{t}\}_{t=1}^{T}$ where $i$ indexes a household with individual history of idiosyncratic shocks $v_{t}$. In Section 4.2, we propose a parametric family of stochastic processes that we later use to approximate the law of motion for these wedges. In Section 4.3 we verify how well our approximations work in practice by performing a similar Monte Carlo analysis on data simulated from the Guerrieri and Lorenzoni (2017) economy. In Appendix B we do the same for the Krusell and Smith (1998) economy. We show there that our approach performs remarkably well in all these examples.

## 4.1 Measuring the wedges from panel data

We assume that we observe a panel of $N$ households’ consumption choices, wages and worked hours, $\{c_{it}, w_{it}, l_{it}\}_{t=1}^{T}$, and the time series for aggregate consumption $\{C_{t}\}_{t=1}^{T}$. We also assume that we have households’ data on financial assets and liabilities. In what follows, we show how we can use these observations to estimate $\{\beta_{i,t+1}, \omega_{t}\}$.

Let us start with $\beta_{i,t+1}$. From equation (19) we have that $\beta_{i,t+1}$ is the conditional expectations, across realizations of the idiosyncratic state in period $t+1$, of the change in the

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14In our application, we will use the Consumption Expenditure Survey (CEX) to obtain these variables.
consumption share of household $i$ between time $t$ and time $t+1$ raised to a power of $-\sigma$. If we were observing multiple realizations of the idiosyncratic shocks for the same household, we could estimate this object by averaging the realized inverse changes in consumption shares between $t$ and $t+1$. In practice, of course, we observe only one realization of consumption choices for each household, and so we need an alternative approach to estimate this conditional expectation.

We proceed as follows. At the beginning of date $t$, we group households according to similar observable characteristics, leaving us with $G$ groups. For each of group $g$, we compute

$$
\beta_{g,t+1} \equiv \frac{1}{N_g} \sum_{j=1}^{N_g} \left[ \left( \frac{c_{jt+1}/C_{t+1}}{c_{jt}/C_t} \right)^{-\sigma} \right], \quad (26)
$$

where $N_g$ is the number of households in group $g$ at time $t$.

The logic behind this approach builds on two premises. The first is that, by grouping individuals along observable characteristics, we are proxying for an individual history $v_t$. The second is that the size of the groups is large enough, so that $\beta_{g,t+1}$ in equation (26) approximates the conditional expectation in equation (19) by the law of large numbers. If these conditions are met, the expression in (26) would be equivalent to the one in (19).

In practice, and because of finite samples, we need to restrict the number of observable characteristics to partition households. In our application, we consider partitions based on the time $t$ value of households’ labor income and financial assets. Specifically, we group households according to whether their labor income at date $t$ is above or below median income and, within each of these two groups, whether their net worth is above or below the group median. Thus, for each $t$, we end up with four different groups of households of approximately equal size: **low income/low net worth**, **low income/high net worth**, **high income/low net worth** and **high income/high net worth**. For each group $g$, we use equation (26) to construct $\beta_{g,t+1}$. The rationale behind this partition is that the current levels of income and net worth are sufficient statistics for an individual history $v_t$ in benchmark models with imperfect risk sharing, for example those that builds on the Bewley-Huggett-Aiyagari framework.

Let us now turn to the measurement of $\omega_t$, defined in equation (20). For each household we first compute the consumption share, $\varphi_{jt} = c_{jt}/C_t$. We then compute $\varphi_{jt}^{\sigma/\psi} e_{it}^{(1+\psi)/\psi}$ and average across $i$

$$
\omega_t = \left[ \frac{1}{N} \sum_{j=1}^{N} \varphi_{jt}^{\sigma/\psi} e_{it}^{(1+\psi)/\psi} \right]^{-1/\psi}. \quad (27)
$$

If $N$ is large enough, this expression is equivalent to the one in (20).
In order to implement the counterfactual described in Section 3.3 we also need to obtain the value of $\omega_t$ in the counterfactual economy with complete markets, see the expression in equation (28). In that case, consumption shares would not be time-varying, but they may be potentially different across households because of initial heterogeneity in wealth. Hence, to compute $\omega_{t}^{cm}$, we need to know the initial distribution of consumption shares and its correlation with $e_{it}$ for every $t$. We assume that the moments of the initial distribution are those of the first year in our sample. That is, we compute $\omega_{t}^{cm}$ as follows

$$\omega_{t}^{cm} = \left[ \frac{1}{N} \sum_{j=1}^{N} \phi_{j1}^{\frac{-e}{\psi}} \times \frac{1}{N} \sum_{j=1}^{N} \epsilon_{jt}^{\frac{1+\phi}{\psi}} + \text{cov} \left( \phi_{j1}^{\frac{-e}{\psi}}, \epsilon_{jt}^{\frac{1+\phi}{\psi}} \right) \right]^{-\frac{1}{\psi}} .$$

(28)

4.2 Stochastic process for the wedges

As in Chari, Kehoe, and McGrattan (2007), we assume a Markov structure for the states, $\Pr(s_t|s_{t-1}) = \Pr(s_t|s_{t-1})$, and suppose that the equilibrium outcome is induced by a recursive competitive equilibrium. Under these assumptions, $\{\beta_{i,t+1}, \omega_{t+1}\}$ are functions of the aggregate state variables of the model. In what follows, we specify a class of stochastic process for $\{\beta_{i,t+1}, \omega_{t+1}\}$ based on a first-order approximation of these functionals.

In a recursive competitive equilibrium, endogenous variables are functions of idiosyncratic and aggregate states. Let $(z, X)$ be the current realization of the aggregate exogenous and endogenous states, with transition $X' = \Gamma(X, z)$, and let $(v, x)$ be the exogenous and endogenous idiosyncratic states of the model, with transition $x' = \gamma(x, v, z, X)$.

To make things more concrete, consider a simple economy nested in the class of models of Section 2. Suppose that households can only save and borrow in the risk-free nominal bond and face a borrowing limit $b_{i,t+1} \geq -\phi$, and assume that their idiosyncratic productivity $e$ is an AR(1) process. In a recursive competitive equilibrium, the exogenous aggregate state is $z = [\theta, A, \epsilon_m]$, the endogenous aggregate state is $X = [\Psi(e, b), i]$—with $\Psi$ being the joint distribution of individual productivities and bond holdings, and $i$ the lagged nominal interest rate—and the idiosyncratic state variables would be $v = e$ and $x = b$.

Due to the recursive structure, it is straightforward to derive the implied stochastic processes for $\{\beta_{i,t+1}, \omega_{t+1}\}$. Specifically, the consumption share of an individual is

$$\phi(v, x, z, X) = \frac{c(v, x, z, X)}{C(z, X)} .$$
Using the above expression, along with the definition of \{\beta_{i,t+1}\} in (19), we can write

\[
\beta_{i,t+1} = \sum_{v_{t+1}} \Pr(v_{t+1}|v_t, z_{t+1}) \left( \frac{\varphi(v_{t+1}, \gamma(v_t, x_t, z_t, X_t), z_{t+1}, \Gamma(z_{t+1}, X_t))}{\varphi(v_t, x_t, z_t, X_t)} \right)^{-\sigma} = f_{\beta_i}(X_t, z_t, z_{t+1}).
\]

Similarly, we can see that in a recursive competitive equilibrium \omega_{t+1} is only a function of the aggregate states at \( t + 1 \), \omega_{t+1} = f_{\omega}(X_t, z_t, z_{t+1}).

Letting \( \hat{y}_t \) be the log-deviation of variable \( y_t \) from its steady state, we have that up to a first-order approximation the law of motion for \( T_{t+1} = [\hat{\beta}_{1,t+1}, \hat{\beta}_{2,t+1}, \ldots, \hat{\omega}_{t+1}]' \) in a recursive competitive equilibrium takes the following form

\[
T_{t+1} = A \times \hat{X}_t + B \times \hat{z}_t + C \times \hat{z}_{t+1},
\]

where the matrices \([A, B, C]\) are functions of the primitives of the model. Given a law of motion for \( z_t \) and \( X_t \), we could then use equation (29) to form expectations over the wedges and to solve for the behavior of aggregate variables using the RA representation.

While (29) gives us the correct law of motion for the wedges up to a first-order approximation, there is a practical hurdle in using it: certain elements of \( z_t \) and \( X_t \) may not be defined in the RA representation. Going back to the example discussed earlier, in the heterogeneous agent economy the distribution \( \Psi(e, b) \) is a state variable. However, this distribution is not defined in the RA representation as it indirectly affects aggregate variables through its effects on the wedges.

To overcome this issue, we partition \( z_t \) and \( X_t \) as follows: \( z_t = [z_{t}^{RA}, z_{t}^{HA}] \) and \( X_t = [X_{t}^{RA}, X_{t}^{HA}] \), where \( (z_{t}^{RA}, X_{t}^{RA}) \) denote the aggregate states that are also defined in the RA formulation of the model while \( (z_{t}^{HA}, X_{t}^{HA}) \) are aggregates states in the heterogeneous-agents economy that are not directly present in the RA representation, i.e. their impact on macroeconomic variables occurs through the wedges. In our applications, we approximate (29) with the following law of motion for \( T_{t+1} \)

\[
T_{t+1} = \Phi(L) \times T_t + A \times \hat{X}_t^{RA} + B \times \hat{z}_t^{RA} + C \times \hat{z}_{t+1}^{RA} + \varepsilon_{t+1},
\]

where \( \Phi(L) \) is a polynomial in the lag operator and \( \varepsilon_{t+1} \) are potentially correlated innovations with variance-covariance matrix \( \Sigma \). Essentially, our approach consists in proxying for the missing state variables in (29) with lagged values of \( T_{t+1} \).16

\[\]
4.3 Monte Carlo analysis

Next, we study whether these approximations work well in practice by performing a Monte Carlo analysis on data simulated from the Guerrieri and Lorenzoni (2017) economy. We focus on their flexible price economy ($\kappa = 0$) where households have the following preferences over consumption and labor,

$$U(c, l) = \frac{c^{1-\sigma} - 1}{1 - \sigma} + \chi \frac{(1 - l)^{1-\psi} - 1}{1 - \psi}.$$  

The functional form for the disutility from labor here is different than the one in our baseline of Section 2, and given these preferences households may choose not to work at a given point in time. Thus, this detailed economy can generate variation in labor supply at the extensive margin. As we shall see, the expression for $\omega_t$ in the RA representation needs to be modified in order to account for this difference.

Households produce a consumption good using the technology,

$$y_{it} = e_{it} l_{it},$$

where $e_{it}$ is an idiosyncratic shock to labor productivity of household $i$ that follows a Markov chain. The households can only save and borrow in a real non-contingent bond earning the risk-free real rate $R_t$ and they face a borrowing limit $b_{i,t+1} \geq -\varphi_t$. The debt limit $\varphi_t$ is the only aggregate shock in this economy, and it follows the AR(1) process,

$$\varphi_t = (1 - \rho) \bar{\varphi} + \rho \varphi_{t-1} + \sigma_{\varphi} \varepsilon_t.$$  

The debt limit $\varphi_t$ is the only aggregate shock in this economy, and it follows the AR(1) process,

In equilibrium, the households’ net demand of bonds equal the supply of bonds by the government, $B$. We follow the authors in the calibration of households’ preferences, the Markov process $e_{it}$, the bond supply $B$ and the average borrowing limit $\bar{\varphi}$. We set $\rho = 0.90$ and $\sigma_{\varphi} = 0.05$. We perform stochastic simulations from this economy by applying the methodology of Boppart, Krusell, and Mitman (2018).

Figure 1 plots the IRFs to a reduction in $\varphi_t$ of two standard deviations—a tightening of the borrowing limit. We can see that the shock leads to a persistent decline in output and in the risk-free rate. The decline in aggregate output is due to a compositional change in labor supply. When the borrowing limit tightens, borrowers cut current consumption while savers increase it. Due to wealth effects, the former increase their labor supply while the latter reduce it. Because borrowers in the model tend to have low-productivity while savers

[17] Since we consider a flexible price economy, we abstract from the specification of monetary policy and nominal variables.
Notes: The figure reports the response of output and the risk-free rate after a tightening of the borrowing limit. Output is reported in percentage deviations from its steady state while the risk-free rate in percentage points.

high-productivity, these changes in labor supply lead to a decline in average productivity, and thus in output. The decline in the risk-free rate is due to the decline in the supply of bonds issued by borrowers.

The RA representation. If aggregate output, \( Y_t \), consumption, \( C_t \), and the risk-free real rate, \( R_t \), are part of an equilibrium of the Guerrieri and Lorenzoni (2017) economy, then, they satisfy the following conditions

\[
\frac{1}{R_t} = \beta \max_{i} E_t \left\{ \beta_{i,t+1} \left( \frac{C_{i,t+1}}{C_t} \right)^{-\sigma} \right\}, \quad (31)
\]

\[
C_t^{-\sigma} = \omega_t \left[ 1 - Y_t \right]^{-\psi}
\]

\[
Y_t = C_t
\]

where \( \beta_{i,t+1} \) is defined in equation (19) and \( \omega_t \) is given by

\[
\omega_t = \left\{ E_t \left[ \left( \frac{\varphi_i \varphi_t^{\psi-1}}{\psi} \right \Phi (l_{i,t} > 0) \right] - \frac{E_t [e_{i,t} \Phi (l_{i,t} > 0)] - 1}{C_t^{\frac{\psi}{\sigma}}} \right\}^{\psi}. \quad (32)
\]

This is the analog of Proposition 1 for this economy. The first equation in (31) is the aggregate Euler equation for bonds. The second equation is obtained from aggregating the labor supply conditions of households that are working, and using the production function to substitute hours worked in efficiency units for aggregate output. This is the analog of the Phillips curve when there are no nominal rigidities, \( \kappa = 0 \). The third equation is the resource constraint.

The variable \( \omega_t \) represents the time-varying disutility of labor in the RA representation and it captures the compositional changes in labor supply that take place in the heterogenous
agent economy. Because there are no price rigidities and no capital, aggregate output and consumption are determined statically using the last two equations in (31), and they are only functions of $\omega_t$. Holding $\omega_t$ constant, fluctuations in $E_t[\beta_{i,t+1}]$ do not have any effect on output, and are fully reflected on the risk-free rate.

In a recursive competitive equilibrium of the heterogeneous agent economy, the exogenous aggregate state is $z_t = z_t^{HA} = [\phi_t]$, and there are no exogenous states that are directly part of the RA representation, $z_t^{RA} = \emptyset$. The endogenous aggregate state is $X_t = X_t^{HA} = \Psi_t(e,b)$ where $\Psi_t$ is the joint distribution of individual productivities and of asset holdings, and there are no endogenous states that are directly part of the RA representation, $X_t^{RA} = \emptyset$.

Equation (30) in this economy specializes to

$$T_{t+1} = A + \Phi(L) \times T_t + \varepsilon_{t+1},$$

(33)

a VAR process.

Monte Carlo analysis. We simulate 500 panel datasets from the original heterogeneous agent economy, with each panel having 10000 households and 100 quarters. We compute $\beta_{g,t+1}$ for the four households’ groups using equation (26), and we compute $\omega_t$ using equation (32). The two high income groups are the ones with the highest average $\beta_{g,t+1}$ in our simulations and the Euler equation errors for those groups are close to zero on average, suggesting that the debt limit doesn’t typically bind for these households. Thus, we include in $T_{t+1}$ only the $\beta_{g,t+1}$ for the high income-high net worth group in order the minimize the number of parameters to be estimated in the law of motion for $T_{t+1}$.

For each panel dataset, we estimate the VAR process in equation (33), setting the lag structure to 1. Given the estimated parameters, we solve for the policy functions of the RA representation using a first-order approximation of the equilibrium conditions in (31). This allows us to compute simulations from the RA representation for each Monte Carlo replication. Panel (a) of Table 1 reports the estimates of the VAR process. We can see that $\omega_t$ is on average positively autocorrelated ($\phi_{\omega,\omega} = 0.54$), and it predicts high values for $\beta_{i,t+1}$, as $\phi_{\beta,\omega} = 0.61$. Given this cross-correlation structure, the RA representation generates persistence in output and in the risk-free rate, and a positive comovement between these two series.\(^{18}\) As we have seen in Figure 1, these are the two key features of the IRFs in the original heterogeneous agent economy. Panel (b) of the table reports first and second moments for output and the risk-free rate in the heterogeneous agent economy along with the same moments computed in the RA representation. Specifically, the table reports the

\(^{18}\)From the system in (31) we can easily see that a high value of the disutility of labor $\omega_t$ reduces output and a high value of $E_t[\beta_{i,t+1}]$ reduces the risk-free rate.
average across these Monte Carlo replications along with the 5th and 95th percentile. We see that the RA representation reproduces very accurately the underlying stochastic behavior of output and the risk-free rate in the heterogeneous agent economy, thus validating our approximations in practice.

5 An application to the US economy

In this section we use the Consumption Expenditure Survey (CEX) to measure the time path of β_{g,t}, \omega_t and \omega_{t}^{cm}. These time series are used, along with the RA representation, to quantify the macroeconomic effects of imperfect risk sharing during the US Great Recession. Section 5.1 presents the data and discusses the time series behavior of the measured \beta_{g,t}, \omega_t and \omega_{t}^{cm}. In Section 5.2 we jointly estimate the stochastic process for these wedges and the structural parameters of the RA representation. Section 5.3 uses the estimated model to asses the impact of imperfect risk sharing for the business cycle, while Section 5.4 performs an event study of the Great Recession.

5.1 Measuring the wedges

In order to compute \{\beta_{g,t}, \omega_t, \omega_{t}^{cm}\}, we need information on changes in households’ consumption shares, (c_{i,t-1}/C_{t-1})/(c_{i,t}/C_t), and on the relative wage per hour worked, e_{i,t} =
We use the CEX to collect information on income, expenditures, employment outcomes, wealth and demographic characteristics for a panel of US households selected to be representative of the population. Households report information on consumption expenditures for a maximum of four consecutive quarters, income and employment information is collected in the first and last interview, and wealth information in the last interview only.¹⁹

The baseline sample includes all households where the head of the household is between the ages of 22 and 64. Our measure of consumption is dollar spending on non-durables and services by the household. We measure wage per hour worked by taking the ratio of labor income to total hours worked. Labor income is a pre-tax measure, and it includes wages and salaries, bonuses, overtime, tips plus income from a business, while total hours worked include hours worked by the head of household and the spouse over the entire year in all jobs. In addition, we obtain socio-demographics indicators about the households (education, sex, family size, etc.) and information on assets and liabilities. The precise definition of the variables is in Appendix C. There, we also discuss in details the sample restrictions we adopt in the analysis.

The model of Section 2 abstracts from important features of the micro data, such as demographics and life-cycle dynamics. In order to have a clear mapping between model and data, we use panel regressions to partial out the effects of these possible confounders. Let \( \tilde{c}_{it} \) be the log of consumption expenditures and \( \tilde{y}_{it} \) the log of labor income. We estimate the following linear equation

\[
\tilde{c}_{it} = \alpha + \gamma' X_i + \gamma y_{it} + e_{it},
\]

where \( X_i \) includes dummies for the sex, race, education, age of the head of household and the state of residence. After estimating this regression, we predict consumption only using labor income and the residual,

\[
\tilde{c}_{it}^p = \alpha + \gamma y_{it} + e_{it}.
\]

We repeat this procedure for all variables used in the analysis.²⁰ After estimating these relationships, we divide all variables in levels, with the exception of wage per hour, by the number of family members in order to obtain per-capita figures.

Appendix C presents summary statistics of the underlying micro data. Households’ characteristics in our sample are comparable with the ones reported in Heathcote and Perri (2018). In line with their findings, we also verify that the behavior of aggregate consumption

¹⁹The CEX asks questions about how assets and liabilities have changed in the preceding year, which allows us to back-date wealth information. See https://www.bls.gov/opub/mlr/2012/05/art3full.pdf for more details.

²⁰We include labor income in the Mincer regressions because we do not want demographic factors to proxy for the effect that idiosyncratic income shocks have on consumption. We do not include \( \tilde{y}_{it} \) when predicting labor income.
expenditures, labor income and hours worked implied by the CEX tracks the corresponding national statistics reasonably well. We finally compute a set of cross-sectional statistics and show that their behavior over time closely mirrors results reported in previous papers in a large body of work on consumption and income inequality (Blundell, Pistaferri, and Preston, 2008; Krueger and Perri, 2006; Aguiar and Bils, 2015; Attanasio and Pistaferri, 2016).

The time path of the wedges. We now use the data to obtain an empirical counterpart to the discount factor wedge. For the subsequent analysis, we will set \( \sigma = 1 \) and \( \psi = 1 \). Following the approach described in Section 4.1, we group households according to whether their income at date \( t - 1 \) is above or below median income and, within each of these two groups, weather the level of their net worth (assets minus liabilities) is above or below the group median. Thus, for each \( t - 1 \), we end up with four different groups of households of approximately equal size. For each group \( g \), we use equation (26) to construct \( \beta_{g,t} \).

We measure \( \beta_{g,t} \) at annual frequency. Following Vissing-Jørgensen (2002), we compute the semi-annual consumption change for each household in our dataset,

\[
\frac{c_{m} + c_{m+1} + c_{m+2} + c_{m+3} + c_{m+4} + c_{m+5}}{c_{m+6} + c_{m+7} + c_{m+8} + c_{m+9} + c_{m+10} + c_{m+11}}
\]

and scale it by an equivalent semi-annual change in aggregate consumption over the same horizon constructed using monthly data of aggregate consumption of non-durable and services from NIPA. We square the resulting ratio to obtain an annualized change. In order to aggregate up to an annual frequency we must allocate this to a given year. This is easiest when the CEX interview aligns perfectly with the calendar year (that is, \( m = 1 \)). In this case we assign this observation solely to this year. However, this case only happens 1/12 of the time. For the rest of the cases we assign the observation to a given year in proportion to its time in that year, e.g. a household whose last interview was in month 7 of year \( t \) would receive a weight of \((7/12)\) in year \( t \) and a weight of \((5/12)\) in year \( t - 1 \).

Figure 2 plots the time path of \( \beta_{g,t} \) for each group along with the year-by-year value of the maximum across the four groups. We normalize this latter to have a mean of 1, and we normalize the other statistics relative to this value. There are two important facts that we

\footnote{In principle, one could consider finer partitions of the joint distribution of income and net worth. However, given the sample size in the CEX, this would produce substantially noisier estimates of \( \beta_{g,t} \). With our partitions, we have roughly 600 households on average per year within each group. In Appendix C.5 we consider alternative partitions including four income groups, two total asset and two income groups, two liquid asset and two income groups. See Figure A-4 for details.}

\footnote{As we discuss later, the level of the wedges is not well estimated in presence of classical measurement errors. Given the prevalence of measurement errors in survey data, we decided to normalize in our analysis the level of the wedges and focus only on their time-variation. This does not have implications for answering our question, as we are interested in gauging the implication of these wedges for business cycle fluctuations.}
Figure 2: The time path of $\beta_{g,t}$ by income and net worth

Notes: Each panel shows an estimate of $\beta_{g,t}$ for four groups of roughly equal size: low income/low net worth, low income/high net worth, high income/low net worth and high income/high net worth (red dashed line) along with the year-by-year value of the maximum across the four groups (solid black line) over the periods 1992-2017. The red dotted lines are 90% confidence bands.

want to emphasize.

First, the high income households have higher $\beta_{g,t}$ relative to low income households for most of the sample. This is shown in the bottom two panels of Figure 2, as the $\beta_{g,t}$ for the two high income groups is equal to the year-by-year max value across all groups (solid line) for most of the years. Through the lens of our framework, this suggests that high income households are the financially unconstrained group—those that attain the maximum in equation (21). Second, $\beta_{g,t}$ for high income households display a substantial increase at the beginning of the Great Recession.

We now turn to the measurement of $\omega_t$ and $\omega_t^{cm}$ in equation (27) and (28). For each household in our panel, we compute the consumption share, $\phi_{i,t} = c_{i,t}/C_t$, and combine it with the relative wage, $e_{i,t} = \frac{w_{i,t}}{W_t}$, to construct these two wedges. Panel (a) in Figure 3 plots the time series for $\omega_t$ and $\omega_t^{cm}$. Both series display a downward trend for most of the sample. Panel (b) of the figure shows that this pattern is explained by the increase in the cross-sectional variance of relative wages a well established fact for the US economy, see
Heathcote, Perri, and Violante (2010). As we explained in Section 3.2, an increase in the cross-sectional variance of labor productivity induces a change in the composition of the labor force that is captured by a lower disutility of labor in the RA representation. We can also observe from the figure that the deviations between $\omega_t$ and $\omega_t^{cm}$ are typically small. This suggests that imperfect risk sharing per se has limited effects on aggregate labor supply through compositional changes in the labor force.

**Measurement errors.** As for all the analysis on households’ consumption based on US surveys, a possible concern is that the fluctuations we measure in the wedges are due to measurement error. While we cannot rule out arbitrary measurement error, in this section we try to gauge whether our results can be attributed to specific forms of measurement error.

One form of measurement error is simply recording errors in the CEX that create extreme outliers. This could be particularly relevant for our analysis because errors in (log) levels of consumption are magnified when they are differenced. When selecting the sample, we remove the top and bottom 1% (year by year) of the observations for all the variables used in the analysis. In addition, we follow Vissing-Jørgensen (2002) and remove observations in consumption growth that are less than 0.20 and greater than 5 (this removes 10 observations).

The second way we address potential measurement error in consumption growth is by adopting the approach in Vissing-Jørgensen (2002) of using semi-annual changes in order to minimize time aggregation and category switching concerns due to the fact that households may only purchase certain categories of goods infrequently.

Third, it is worth noting that the wedges are cross-sectional averages of household-level observations, and the process of averaging by itself can reduce the impact of measurement errors.
errors. To see why, suppose that observed consumption is related to the true consumption, \( \tilde{c}_{j,t} \), as follows \( c_{j,t} = \tilde{c}_{j,t} \times \exp\{\eta_{j,t}\} \) where \( \eta_{j,t} \) is a Gaussian iid measurement error with mean \(-\sigma^2_\eta/2\) and variance \( \sigma^2_\eta \). Assume further that \( \eta_{j,t} \) is independent from the true level of consumption \( \tilde{c}_{j,t} \). Then, we have

\[
\frac{1}{N_i} \sum_{j=1}^{N_i} \frac{c_{j,t-1}}{c_{j,t}} = \frac{1}{N_i} \sum_{j=1}^{N_i} \exp\{-\Delta \log(\tilde{c}_{j,t})\} \times \frac{1}{N_i} \sum_{j=1}^{N_i} \exp\{-\Delta \eta_{j,t}\}
+ \text{Cov}\left(\exp\{-\Delta \log(\tilde{c}_{j,t})\}, \exp\{-\Delta \eta_{j,t}\}\right).
\]

If \( N_i \to \infty \), then the covariance term goes to zero and the measured \( \beta_{i,t} \) becomes

\[
\beta_{i,t} = \tilde{\beta}_{i,t} \times \exp\{\sigma^2_\eta\},
\]

where \( \tilde{\beta}_{i,t} \) is the statistics computed using the true consumption. That is, in the case of classical measurement errors, \( \beta_{i,t} \) is off relative to the truth by a time-invariant factor. A similar derivation can be done for \( \omega_t \). Because our analysis is not focused on the levels of these variables but rather on their changes over time, it is robust to the presence of classical multiplicative measurement errors in consumption and relative wages.

Fourth, as we will see in the next subsection, we model explicitly a measurement error for both wedges when estimating their stochastic process. This is intended to mitigate the impact that non-classical measurement errors on households data has on the wedges.

5.2 Estimating the RA representation

The vector of exogenous states in the RA representation is \( z_t^{RA} = [\tilde{\theta}_t, \tilde{A}_t, \epsilon_{m,t}] \). We assume that the aggregate preference and technology shocks in logs follow independent AR(1) processes,

\[
\tilde{\theta}_t = \rho_{\theta} \tilde{\theta}_{t-1} + \epsilon_{\theta,t} \quad \epsilon_{\theta,t} \sim \mathcal{N}(0, \sigma^2_{\theta})
\]

\[
\tilde{A}_t = \rho_a \tilde{A}_{t-1} + \epsilon_{A,t} \quad \epsilon_{A,t} \sim \mathcal{N}(0, \sigma^2_{A}),
\]

and that monetary policy innovations are Gaussian, \( \epsilon_{m,t} \sim \mathcal{N}(0, \sigma^2_m) \). The endogenous state variables are the nominal interest rate and the wedges, \( T_t \). The vector of wedges contains \( \omega_t, \omega_t^{cm} \) and only the \( \beta_{g,t} \) for the group with the highest average value for this wedge in our sample, the high income/low net worth group.\(^{23}\) These variables are expressed in logs and

---

\(^{23}\)The implicit assumption is that the group with the highest average \( \beta_{g,t} \) over the sample is comprised of households that are financially unconstrained and that realization of the discount factor wedge for the other groups does not have further predictive power for \( \beta_{g,t+1} \). In the Monte Carlo analyses of Appendix B we show that this assumption works well when simulating data from benchmark incomplete market models.
de-meaned, \( T_t = [\hat{\beta}_t, \hat{\omega}_t, \hat{\omega}_m] \). The law of motion for \( T_t \) is based on equation (30). We restrict \( \Phi(L) \) to have a one-lag structure and to be block diagonal, so that \( \hat{\omega}_m \) depends only on its own lags and does not load on the other equations. The other parameters in (30) are left unrestricted.

Along with the parameters for these stochastic processes, we need to specify numerical values for the parameters governing preferences, \( [\beta, \sigma, \psi, \chi] \), price adjustment costs, \( \kappa \), the elasticity of substitution across varieties, \( \mu \), and the Taylor rule, \( [\rho_i, \gamma_{\pi}, \gamma_y, \pi^*] \). We fix a subset of these parameters to conventional values in the literature. Consistent with the measurement of the wedges, we set \( \sigma = \psi = 1 \). We let \( \mu = 1.2 \) and set \( \chi \) to \( 1/\mu \), so that consumption and output equal 1 in a deterministic steady state of the model. Finally, we set the target inflation rate to 2%, and \( \beta = 0.99 \), values that guarantee that the model matches the average inflation and nominal interest rate in our sample in a deterministic steady state.

The remaining model parameters are estimated with Bayesian methods using annual data for the 1992-2017 period. As it is customary in the literature, we estimate the model using observations on output, nominal interest rates and inflation, to which we add the wedges. Thus, the vector of observable variables is \( Y_t = [\hat{Y}_t, i_t, \pi_t, T_t] \), and we denote by \( Y_T \) all the observations in our sample. The state vector is \( S_t = [i_{t-1}, \hat{\theta}_t, \hat{A}_t, \varepsilon_m, T_t] \). The RA representation of Proposition 1 defines the non-linear state space model

\[
Y_t = g(S_t; \phi) + \eta_t \\
S_t = f(S_{t-1}, \varepsilon_t; \phi),
\]

where \( g(\cdot) \) and \( f(\cdot) \) represent the policy functions of the model, \( \phi \) the vector of parameters to be estimated, \( \varepsilon_t \) collects the innovations to the stochastic variables of the model. Appendix D describes the numerical algorithm we use to solve for the decision rules in presence of an occasionally binding zero lower bound constraint on nominal interest rates. The vector \( \eta_t \) collects Gaussian measurement errors that capture deviations between the data \( Y_t \) and \( g(S_t; \phi) \). We introduce measurement errors only for the wedges, as those are measured using survey data, and fix their variance to 10% of the unconditional variance of these series.

Given this representation, we can apply filtering techniques to the state-space system and evaluate the likelihood of the model, \( L(\phi|Y^T) \). We can then combine the likelihood function with a prior for the structural parameters, \( p(\phi) \), and apply the Metropolis-Hastings

---

24Note that \( \hat{\omega}_m \) does not affect endogenous variables in the RA representation. However, we include it in \( T_t \) so that we can estimate its stochastic process. This will be needed later in the analysis to compute counterfactuals.

25We map the log of output (net of adjustment costs), \( \hat{Y}_t \), to the percentage deviations of log real GDP from a linear deterministic trend. The inflation rate \( \pi_t \) is the annual percent change in the consumer price index, and \( i_t \) is mapped to the annual effective federal funds rate.
algorithm to sample from the posterior distribution of $\phi$ (An and Schorfheide, 2007), see Appendix E for some details. For the purpose of estimation, we solve for the policy functions of the model with a first-order perturbation. The first-order perturbation solution is much faster and numerically more stable than the global approximation discussed in Appendix D, and it allows us to use the Kalman filter for the evaluation of the likelihood function. The main drawback is that, by using perturbation methods, we do not account in the estimation for the possibility of a binding zero lower bound constraint on nominal interest rates. However, as we will see in the next sub-section, the non-linear model fits the data remarkably well once we apply the estimates that we obtain here. Thus, we believe we would obtain very similar estimates for the model parameters if we were to estimate it with non-linear methods.

Table A-3 and A-4 in Appendix E reports prior and posterior statistics for the model’s parameters. The estimates for the parameters of the Taylor rule and of the price adjustment costs are in line with previous estimates reported in the literature. For instance, they are comparable to the ones in the working paper version of Gust, Herbst, López-Salido, and Smith (2017), where the authors estimate a representative agent three-equations New Keynesian with technology shocks, discount factor shocks and monetary policy shocks. Our prior on the stochastic process of the wedges is that $[\hat{\beta}_t, \hat{\omega}_t, \hat{\omega}_c^{cm}]$ follow three independent AR(1) processes. Posterior statistics look quite different from this benchmark. In particular, a low realization of the TFP shock or a high realization of $\hat{\theta}_t$ predict a significantly higher level of the discount factor wedge, $E_t[\hat{\beta}_{t+1}]$. This behavior is consistent with that of structural models in which idiosyncratic income risk and/or households’ inability to smooth these shocks is more prevalent during recessions. The monetary policy shock, instead, does not affect significantly $E_t[\hat{\beta}_{t+1}]$. Finally, note that the parameters of the stochastic process for $\hat{\omega}_c^{cm}$ are close to those of $\hat{\omega}_t$. This is not surprising given that, as we showed earlier, the two series have very similar behavior over the sample period.

5.3 Imperfect risk sharing and the business cycle

Table 2 reports first and second moment for output, inflation and nominal interest rate in the data and in the estimated RA representation. These three variables are positively correlated and the estimated model fits well this pattern. In addition, the estimated model captures well the volatility and persistence of these three series and produces a fairly realistic frequency for episodes with a binding zero lower bound constraint.

---

26To construct the latter, we solve for the policy functions of the RA representation numerically using the algorithm described in Appendix D, and use the policy functions to simulate the model. The moments are then computed on simulated data. For this experiment, we fix the structural parameters at the posterior mean reported in Table A-3.
Table 2: Sample statistics: model vs. data

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RA representation</th>
<th>Counterfactual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean($\pi_t$)</td>
<td>2.29</td>
<td>1.94</td>
<td>2.14</td>
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<tr>
<td>Mean($i_t$)</td>
<td>2.67</td>
<td>3.21</td>
<td>3.38</td>
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<tr>
<td>Stdev($Y_t$)</td>
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<td>Stdev($\pi_t$)</td>
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<td>Stdev($i_t$)</td>
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<td>2.28</td>
<td>1.82</td>
</tr>
<tr>
<td>Corr($Y_t, Y_{t-1}$)</td>
<td>0.94</td>
<td>0.78</td>
<td>0.77</td>
</tr>
<tr>
<td>Corr($i_t, i_{t-1}$)</td>
<td>0.24</td>
<td>0.57</td>
<td>0.44</td>
</tr>
<tr>
<td>Corr($\pi_t, \pi_{t-1}$)</td>
<td>0.85</td>
<td>0.79</td>
<td>0.84</td>
</tr>
<tr>
<td>Corr($Y_t, i_t$)</td>
<td>0.47</td>
<td>0.44</td>
<td>-0.03</td>
</tr>
<tr>
<td>Corr($Y_t, \pi_t$)</td>
<td>0.59</td>
<td>0.16</td>
<td>-0.33</td>
</tr>
<tr>
<td>Corr($i_t, \pi_t$)</td>
<td>0.52</td>
<td>0.65</td>
<td>0.62</td>
</tr>
<tr>
<td>% time with binding ZLB</td>
<td>0.19</td>
<td>0.13</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Notes: Data statistics are computed over the 1984-2017 period. Output, inflation and nominal interest rates are reported in percentages. Sample statistics for the RA representation and the complete markets counterfactual are computed on a long simulation ($T = 50,000$) from these two economies.

The third column in Table 2 reports the same statistics for a counterfactual economy with complete financial markets. This economy is identical to the RA representation, with the exception that the wedges are set to their counterfactual value under complete markets, $\hat{\beta}_t = 0$ and $\hat{\omega}_t = \hat{\omega}^{cm}_t$. The comparison between column two and three is thus informative about the macroeconomic implications of imperfect risk sharing: if shocks and frictions at the micro level were an important source of business cycle fluctuations, we should expect that the introduction of complete financial markets would sensibly reduce the volatility of output. Because the estimated stochastic process for $\hat{\omega}^{cm}_t$ is very similar to that of $\hat{\omega}_t$, the differences across these two economies are mostly due to the time-variation in $E_t[\hat{\beta}_{t+1}]$ that take place in the RA representation but not in the economy with complete markets.

We can see from Table 2 that the complete markets economy is characterized by a smaller standard deviation for inflation and nominal interest rates relative to the RA representation, 60% and 80% respectively. However, the standard deviation of output across the two economies is fairly comparable: imperfect risk sharing adds only 7% to output volatility relative to its complete financial markets equivalent.

Why do we find that imperfect risk sharing on average does not have sizable effects on output dynamics? Since $\hat{\omega}_t$ and $\hat{\omega}^{cm}_t$ have similar stochastic processes, the key channel through which these frictions impact the aggregate are isomorphic to a time-varying discount rate in an otherwise standard three-equations New Keynesian model. For a fixed interest rate, an increase in the discount factor leads the stand-in household to reduce its demand for the
final good. The monetary authority, however, reacts to the shock by cutting interest rates, dampening the impact of the increase in patience on consumption and output. Given our estimates of the Taylor rule, the response of the monetary authority is strong enough to offset most of the output effects of changes in $E_t[\hat{\beta}_{t+1}]$.

Importantly, the above logic depends on the response of the monetary authority. It is well known in the literature that changes in the discount factor can have substantial output effects when the zero lower bound constraint on nominal interest rate binds, as in that case the monetary authority cannot cut further nominal interest rates. These events are somewhat rare in the estimated model, because the zero lower bound binds only 13% of the times. Thus, while the results of Table 2 indicate limited output effects of imperfect risk sharing on average over the business cycle, they do not rule out that these frictions may have more sizable effects in periods during which the monetary authority is constrained by the zero lower bound. In the next subsection we explore this possibility with an event study of the US Great Recession.

### 5.4 Imperfect risk sharing and the US Great Recession

We now use the estimated model to measure the macroeconomic effects of imperfect risk sharing during the Great Recession. To this end, we proceed in two steps. We first apply the particle filter to the state-space model (34) in order to estimate the realization of the structural shocks that rationalize the time path of observable variables during this event. We next feed the structural shocks in the economy with $\hat{\beta}_t = 0$ and $\hat{\omega}_t = \hat{\omega}_t^{cm}$ in order to generate the counterfactual paths for output, inflation and nominal interest rates under complete financial markets. The difference between what we observe in the data and these counterfactual paths isolates the macroeconomic effects of imperfect risk sharing during the Great Recession. Appendix E provides a detailed description of both steps.

Starting with the first step of this procedure, Figure 4 reports the data (circled lines) along with the posterior mean (solid line) and 90% credible set for their model counterpart. The figure also reports the estimates for the three latent structural shocks. By construction, the model tracks very closely output, inflation and nominal interest rates during the event because of the absence of measurement errors on these variables in the state space representation (34), and it fits well the behavior of the wedges $[\hat{\beta}_t, \hat{\omega}_t]$.

In order to replicate these paths, the model infers a substantial increase in the discount factor of the stand-in household: the combined discount factor $E_t[\hat{\beta}_{t+1} + \hat{\theta}_{t+1}]$ increases by four percentage points between 2008 and 2010. This is a well known result from the literature: the canonical New Keynesian model needs an increase in the discount factor in order to fit the fall in output, inflation and nominal interest rate observed during the Great Recession.
Recession. Interestingly, $E_t[\hat{\beta}_{t+1}]$ accounts for two of the four percentage points increase that is needed to fit the aggregate data. That is, we estimate that imperfect risk sharing accounts for roughly half of the increase in the discount rate that is needed by the prototypical New Keynesian model to fit the Great Recession.

Equipped with the path $S_t$, we can then construct the trajectories for output, nominal interest rates and inflation that would prevail in an economy with complete financial markets. For that purpose, we solve numerically for the policy functions of the RA representation with $\beta_t = 1$ and where $\hat{\omega}_t = \hat{\omega}^{cm}_t$. We then construct the counterfactual paths by feeding these policy functions with the estimated path for $\{\hat{\theta}_t, \hat{A}_t, \epsilon_{m,t}\}$ and $\hat{\omega}^{cm}_t$, see Appendix E for a detailed description of this experiment.

Figure 5 compares the trajectories for output, inflation and nominal interest rates in this counterfactual (circled lines) with the actual trajectories in US data (solid lines) during the 2007-2011 period. From 2007 to 2009-2010, de-trended real GDP in the US fell by 8%. The counterfactual economy with complete financial markets displays a smaller decline in real economic activity during this period, respectively 5.5% and 6.0%. Thus, our findings are consistent with the view that deviations from perfect risk sharing were an important dimension of the US Great Recession, accounting for roughly one-fourth of the observed fall in
Why do we observe these differences between the baseline and the counterfactual economy? From Figure 4 we can see that \( E_t[\hat{\beta}_{t+1}] \) contributes substantially to the increase in the overall discount rate during the Great Recession. As nominal interest rates were at zero in 2009-2010, these developments led to a substantial decline in aggregate demand and inflation. In the complete markets economy, \( \hat{\beta}_t \) is not time-varying, and the counterfactual features substantially output, inflation and nominal interest rates.

It is important to stress that the quantification of the aggregate effects in this section depends on the assumptions we made about production, nominal rigidities and the conduct of monetary policy. We performed the analysis within the context of the “three-equations” model with price rigidities because it is transparent and it is the core of the vast majority of medium-scale New Keynesian models studied in the literature. As we explained earlier, it is relatively straightforward to modify these assumptions and apply our methodology in more complex models. For example, in a previous version of the paper we considered a variant of this model with capital accumulation and verified the robustness of our findings.
6 Inspecting the mechanism

The main result from the previous section is that one-fourth of the fall in output during the Great Recession can be attributed to imperfect risk sharing. A natural question at this stage is what economic mechanisms can account for this result. The approach we have developed in this paper is mostly silent about this matter: precisely because the RA representation of Proposition 1 holds for every model in the class that we consider, two economies with different primitives could in principle generate the same patterns for \( \{ \beta_i, \omega_t \} \). The objective of this section is to go beyond the RA representation and use additional information to address this question.

A first useful exercise consists in understanding which moment of the distribution of households’ consumption shares explains the increase in \( \beta_{g,t} \) that we observed during the Great Recession. We focus on this wedge because it played a major role in the counterfactual of the previous section. Specifically, we can decompose \( \beta_{g,t} \) defined in (26) as follows

\[
\beta_{g,t} = \left[ \frac{C_{g,t} / C_t}{C_{g,t-1} / C_{t-1}} \right]^{-1} \left[ \frac{1}{N_g} \sum_{j=1}^{N_g} \left[ \frac{c_{g,t} / c_{g,t-1}}{c_{j,t} / c_{j,t-1}} \right] \right] \beta_{AVG,gt} + \beta_{JEN,gt},
\]

(35)

where \( C_{g,t} \) is the average consumption of group \( g \), \( C_{g,t} = (1/N_g) \sum_{j=1}^{N_g} c_{j,t} \). Mechanically, \( \beta_{g,t} \) can increase for two reasons. First, if the average consumption share of households in group \( g \) falls over time. This effect is captured by the term \( \beta_{AVG,gt} \) in equation (35). Second, because of Jensen’s inequality, an increase in \( \beta_{g,t} \) can occur for a change in higher moments of the distribution of consumption share—for instance, an increase in the cross-sectional dispersion of \( c_{jt} / c_{jt-1} \). These effects are captured by the term \( \beta_{JEN,gt} \) in (35).

This decomposition is useful for model discrimination, as two models that generate the same increase in \( \beta_{g,t} \) may differ on their implications for \( \beta_{AVG,gt} \) and \( \beta_{JEN,gt} \). The “two-agent” New Keynesian model studied in Galí, López-Salido, and Vallés (2007), Bilbiie (2008), and Debortoli and Galí (2017) emphasizes differential consumption growth between hand-to-mouth agents and savers as the key mechanism through which imperfect risk sharing affect the business cycle. By construction, this model does not generate dispersion in consumption shares for the agents that are unconstrained. Hence, \( \beta_{JEN,gt} \) equals 1 for every \( t \), and all the variation in \( \beta_{g,t} \) arises because of changes in \( \beta_{AVG,gt} \). Models with richer heterogeneity can instead generate time-variation in \( \beta_{JEN,gt} \).

Figure 6 reports this decomposition during the Great Recession for both of the high income groups. There are two key takeaways. First, \( \beta_{g,t} \) rises significantly during the Great Recession in each group. Second, while both components increase, the Jensen inequality
term accounts for between 1/2 to 2/3 of the increase in 2008/2009 in $\beta_{g,t}$. That is, much of the increase in the measured discount factor during the Great Recession is due to an increase in the dispersion of consumption changes for the high income groups. These results suggest that idiosyncratic risk within these groups is of first-order importance for understanding the Great Recession episode.

Within the class of models in the Bewley-Hugget-Aiyagari tradition, an increase in $\beta_{JEN,g,t}$ for unconstrained households can come from two sources. First, and fixing the set of risk-sharing mechanisms available to households, we may observe an increase in the dispersion of consumption shares for unconstrained agents if their idiosyncratic labor income risk increases. This is the mechanism emphasized, among others, by Bayer et al. (2019) and Heathcote and Perri (2018). Second, and fixing idiosyncratic income risk, an increase in $\beta_{JEN,g,t}$ for unconstrained households may arise because of a deterioration of their risk-sharing mechanisms. For example, a persistent tightening of borrowing constraints makes future consumption more sensitive to negative income shocks even for households that are currently not constrained. This mechanism is operating in the Guerrieri and Lorenzoni (2017) economy.

We can use CEX data to investigate which of these two mechanisms can better account for the increase in dispersion in Figure 6. A first direct check is to see whether there has been a change in the distribution of income changes during the Great Recession for the high

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27 This can be seen by applying the decomposition in (35) to the example of Section 3.2.
income groups, the ones that represent financially unconstrained households. The left-panel in Figure 7 displays a histogram of $\frac{y_t}{y_{t-1}}$ in the cross-section of households before (2006-07), during (2008-09) and after (2010-11) the Great Recession for the high income group. The histograms from each period are quite similar that there was not a shift in labor income risk for high income households during the Great Recession.\footnote{These results are consistent with previous papers that examined the cyclical behavior of income changes. While Storesletten, Telmer, and Yaron (2004) found evidence of countercyclical income risk, their estimates are obtained from an estimated income process for the entire population, and it is not comparable to Figure 7 that condition on households with current income above the median. Heathcote, Perri, and Violante (2010) find little cyclical variation in earnings growth for households between the 50th and 90th percentiles of the earnings distribution, which corresponds roughly to the sub-group displayed in Figure 7. The authors document significant cyclical variation for poorer households due to a higher incidence of unemployment during recessions, a result that is confirmed by Guvenen, Ozkan, and Song (2014) with Social Security Administration data.}

Figure 7: Income dispersion and the sensitivity of households’ consumption to income changes

Notes: The left panel plots the distribution of income changes ($\frac{y_{it}}{y_{it-1}}$) for the high income households over three time periods: 2006-07, 2008-09 and 2010-11. The right panel plots the coefficient $\gamma$ in equation (36) for both the low income (top panel) and high income groups.

Second, we check if there has been an increase in the sensitivity of consumption to income changes during the Great Recession. Specifically, we estimate the following linear relation biannually over there period 2006-2011 separately for households with above and below median income:

$$\left( \frac{c_{jt-1}}{C_{t-1}} / \frac{c_{jt}}{C_t} \right) = \alpha + \beta \frac{y_{jt}}{y_{jt-1}} + \delta \text{neg}_t + \gamma \frac{y_{jt}}{y_{jt-1}} \times \text{neg}_t + e_{jt}, \quad (36)$$

where $\text{neg}_t$ is an indicator function equal to 1 if household income is declining, $\frac{y_{it}}{y_{it-1}} < 1$. This specification is motivated by the fact that there is sharp non-linearity in the relationship between changes in consumption shares and income changes depending on whether changes in income are positive or negative. The parameter $\gamma$ captures this differential sensitivity,
with a negative value indicating that households’ consumption shares are more sensitive to negative income changes. The bottom panel in Figure 7 shows that \( \gamma \) declines during the Great Recession for households with high income, suggesting that the increase in the Jensen term of equation (35) for this group arises because of an increase in the sensitivity of consumption shares to income changes. The top panel of Figure 7 documents that there is no change in sensitivity for the low income groups.

The evidence in Figure 7 suggests that structural models that emphasize a reduction in the ability of unconstrained agents to smooth income shocks during the Great Recession have a better chance of being consistent with the micro data than structural models emphasizing an increase in the cross-sectional dispersion of idiosyncratic labor income. A labor-income risk explanation can however be consistent with the evidence in Figure 7 if the Great Recession was associated with a change in the nature of idiosyncratic income shocks affecting high income individuals. For example, suppose that the Great Recession was accompanied by an increase in the variance of the permanent component of income and a reduction of its transitory component.\(^{29}\) Such a shift could explain why in Figure 7 we detect an increase in the responsiveness to negative income shock in absence of significant shifts in the distribution of income changes. This is because permanent shocks are harder to smooth than transitory ones in baseline incomplete markets model, and so consumption is more sensitive to the former than the latter.

7 Conclusion

We have proposed a simple approach to assess the macroeconomic implications of imperfect risk sharing implied by a class of New Keynesian models with isoelastic preferences, idiosyncratic income risk and incomplete financial markets. In this class of models, households’ inability to insure idiosyncratic risk is reflected in time-variation in their consumption shares. Leveraging this insight, we use households’ consumption choices to directly measure the degree of imperfect risk sharing for the US economy. We have documented a deterioration of risk sharing during the US Great Recession, as the cross-sectional dispersion of households’ consumption shares increases during this period. We have then proposed a methodology to quantify the aggregate implications of these movements. Through the lens of a prototypical New Keynesian model, we show that these deviations from perfect risk sharing contributed to roughly one-fourth of the output losses observed in 2009-2010.

Our paper is mostly silent about the shocks and frictions that contribute to the observed

\(^{29}\)This hypothesis is similar to the mechanism in Blundell, Pistaferri, and Preston (2008) to account for the changes in the consumption distribution relative to the income distribution over the 1970s to 1990s.
deviations from perfect risk sharing. However, it clarifies that these different model ingredients matter for aggregate fluctuations only through their impact on two summary statistics of the joint distribution of households’ consumption shares and relative wages, what we labeled preference wedges. These statistics can be computed using panel data, and we believe they should be an important empirical target for researchers that are interested in assessing the implications of imperfect risk sharing for the business cycle. In addition, these statistics can be used to discriminate between various risk sharing mechanisms, in the spirit of Karaivanov and Townsend (2014). We leave this analysis to future research.

References


Appendix to “Imprecise Risk Sharing and the Business Cycle”

by David Berger, Luigi Bocola and Alessandro Dovis

A Additional examples

In this appendix we derive the RA representation for two additional detailed economies. We first consider an economy inspired by Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017). In this environment, the debt limit that households face is stochastic, and we will see that a tightening of this constraint maps into an increase in the discount factor of the RA representation. We next consider an economy along the line of the one studied by Auclert, Rognlie, and Straub (2018), and discuss the relation between the wedges in the RA representation and the notion of intertemporal marginal propensity to consume, which the authors identify as a sufficient statistic for evaluating the effects of fiscal transfers in their model.

A.1 Credit constraints and aggregate demand

Consider now an economy where the debt limit depends on aggregate conditions. Households can only trade in a noncontingent bond in zero net supply subject to a debt limit. To simplify the derivations, we abstract from idiosyncratic uncertainty and consider an economy with different types of agents whose income is determined by the aggregate state of the economy $z_t$. We assume that there are only two types of agents, $i = 1, 2$ of equal measure and the labor efficiency units of type $i$ is $e_i(z_t) \in \{e_L, e_H\}$ with $.5e_H + .5e_L = 1$ and for any $z_t$ if $e_1(z_t) = e_H$ then $e_1(z_t) = e_L$ and vice versa. To simplify the algebra, we further assume that profits from the monopolistic competitive firms are distributed to households so that $T_i(z^t) + w(z^t)e_i(z_t)l_i(z^t) = e_i(z_t)C(z^t)$.

Households can trade only a non-contingent bond in zero-net supply, subject to the debt limit $\phi(z^t)$. Thus, asset holdings must be such that

$$b_i(z^t) \geq -\phi(z^t)$$

In what follows, we assume that $\phi(z^t)$ is sufficiently small so that the debt limit is always binding for the agents with a low realization of the individual productivity shock, $e_L$. Thus, using that $b_i(z^t) = -\phi(z^t)$ if $e_i(z^t) = e_L$ in the budget constraints and the market clearing condition for the bond in zero net supply, the individual consumption allocations are given
by
\[ c_i(z^t) = \begin{cases} 
  e_L C(z^t) + b_i(z^{t-1}) \frac{\phi(z^t)}{1+\pi(z^t)} + \frac{\phi(z^t)}{1+i(z^t)} & \text{if } e_i(z_t) = e_L \\
  e_H C(z^t) + b_i(z^{t-1}) \frac{\phi(z^t)}{1+\pi(z^t)} - \frac{\phi(z^t)}{1+i(z^t)} & \text{if } e_i(z_t) = e_H 
\end{cases} \]  
(A.1)

where \( b_i(z^{t-1}) \) depends on the particular history:
\[ b_i(z^{t-1}) = \begin{cases} 
  \phi(z^{t-1}) & \text{if } e_i(z_{t-1}) = e_H \\
  -\phi(z^{t-1}) & \text{if } e_i(z_{t-1}) = e_L 
\end{cases} \]

Moreover, the Euler equations for bond holdings are
\[ u'(c_i(z^t)) > \beta(1+i(z^t)) \sum_{z^{t+1}} \Pr(z^{t+1}|z^t) \frac{u'(c_i(z^{t+1}))}{1+\pi(z^{t+1})} \]  
if \( e_i(z_t) = e_L \)  
(A.2)
\[ u'(c_i(z^t)) = \beta(1+i(z^t)) \sum_{z^{t+1}} \Pr(z^{t+1}|z^t) \frac{u'(c_i(z^{t+1}))}{1+\pi(z^{t+1})} \]  
if \( e_i(z_t) = e_H \).  
(A.3)

That is, the agent with a high realization of the individual productivity in \( z^t \) has the highest valuation for the bond and attains the maximum in the aggregate Euler equation (21). Using (A.1), we can then express the \( \beta_i(z^{t+1}) \) for this type as
\[ \beta_i(z^{t+1}) = \left( \frac{e_i(z_{t+1}) + \frac{\phi(z_t)}{1+\pi(z^t)} - \frac{b_i(z^{t+1})}{1+i(z^{t+1})}}{e_H(z_t) + \frac{b_i(z^{t-1})}{1+\pi(z^t)} - \frac{\phi(z_t)}{1+i(z^t)}} / C(z^t) \right)^{-\sigma} \]

From the expression above, it is evident how a tightening of the debt limit, a reduction in \( \phi(z^t) \), increases \( \beta_i(z^t, z_{t+1}) \) for the marginal agent. Intuitively, a reduction in \( \phi(z^t) \) means that agent with a low income shock can borrow less to smooth their consumption. In equilibrium, this implies that the agent with currently higher income must save less and consume more to clear the asset market. The increase in current consumption share makes this agent more willing to save and thus the measured \( \beta_i(z^t) \) increases for the marginal agent.

This example illustrates two properties of our equivalent representation. First, the behavior of non-marginal households is not irrelevant for the dynamics of aggregates despite only the consumption profile for agents on their individual Euler equation appearing in the aggregate Euler equation 21. Here the behavior of borrowing constrained households affects the discount factor of the stand-in household through a general equilibrium relationship.

This is important because the literature so far has emphasized the role of these agents with high marginal propensity to consume as critical for the propagation of aggregate shocks. Our representation does not contradict this intuition.
Second, the above expression for $\beta_i(z^t, z_{t+1})$ shows how the “micro-block” is not independent from the “macro-block” that determines the dynamics of aggregates as the $\beta_i(z^t, z_{t+1})$ depends on aggregate consumption, the inflation rate, and the policy rate.

### A.2 Fiscal transfers and aggregate demand

There is a large and growing literature that has emphasized the role of the distribution of marginal propensities to consume (MPCs) as a critical statistic to discipline structural New Keynesian models with heterogeneous agents. This is because MPCs are informative about the response of aggregate variables to redistributive policies or to shocks like the tightening of borrowing constraints in partial equilibrium. Auclert, Rognlie, and Straub (2018) show that a summary statistic for the distribution of MPCs at different time horizons – what they term intertemporal MPCs – is a sufficient statistic for the response of output to fiscal shocks in a class of New Keynesian models. Here we show how our work is connected to this line of research. In particular, we show that there is a mapping between the relevant statistic of the distribution of intertemporal MPCs and the impulse response function to aggregate shocks of the discount factor and disutility of leisure of the stand-in household in our framework.

We do so within the context of a simple two-period economy with shocks to fiscal policy that satisfies the conditions in Auclert, Rognlie, and Straub (2018).

Let $t = 1, 2$ and assume that wages are sticky in period 1 and flexible in period 2. Assume there are two types of agents, $i \in \{L, H\}$. Agents differ only in their endowment of efficiency unit of labor in period 1, $e_H > e_L$. Since wages are sticky in period 1, we need to postulate a mechanism for the allocation of labor between the two types of agents. We assume each agent works the same amount of hours so that the real labor income of type $i$ agent is $e_i Y_1$.

The resource constraints are then

$$Y_1 = \sum_i \lambda_i e_i l_1, \quad Y_2 = \sum_i \lambda_i l_2$$

Fiscal policy consists of a lump-sum transfer in period 1 financed by issuing debt to be repaid in period 2 with lump-sum taxes. For simplicity we allow for taxes in period 2 to depend on the household’s type.

The problem for a household of type $i$ is

$$\max_{c_{1i}, d_i, c_{2i}, l_{2i}} \sum_{t=1}^{2} \beta^{t-1} \left[ \log c_{1i} - \frac{1 + \psi}{1 + \psi} \right]$$

A-3
subject to

\[ c_{1i} + a_i \leq e_i Y_1 + T \]
\[ c_{2i} \leq w_2 l_{2i} - \tau_{2i}(T) + (1 + r)a_i \]
\[ a_i \geq 0 \]

where \( a_i \) are the holdings of government debt, \( T \) are lump-sum transfers in period 1, and \( \tau_i(T) \) are lump-sum taxes in period 2.

The government budget constraint in period 1 is

\[ T = \sum_i \lambda_i a_i \]

and in period 2 is

\[ \sum_i \lambda_i (1 + r) a_i = \sum_i \lambda_i \tau_i(T) \]

To simplify the algebra, we assume that taxes in period 2 are given by \( \tau_L(T) = 0 \) and \( \tau_H(T) = T/\lambda_H \) so that there are no wealth effect in period 2 and both types of agents consume and work the same amount. Finally, we assume that monetary policy targets a real rate \( r \) as Auclert, Rognlie, and Straub (2018).

We can then characterize the equilibrium in this economy as a function of \( T \). In period 2, the output is efficient and it solves \( \frac{1}{Y_2} = \chi Y_2^\phi \). We normalize \( \chi \) to one so that \( Y_2 = c_{2H} = c_{2L} = 1 \).

If \( T \) is small enough, the debt limit \( a_i \geq 0 \) is binding for the type \( L \) agents and the allocations in period 1 are given by

\[ Y_1 = \frac{1}{\beta (1 + r) e_H} + \frac{1 - \lambda_H}{e_H \lambda_H} T \]
\[ c_{1H} = e_H Y_1 - \frac{1 - \lambda_H}{\lambda_H} T \]
\[ c_{1L} = e_L Y_1 + T \]

Thus the effect of an increase in transfers in period 1 is expansionary and given by

\[ \frac{\partial Y_1}{\partial T} = \frac{1 - \lambda_H}{e_H \lambda_H} \]

Consider now our representative agent formulation for this economy. The taste shock for
the marginal agent, $\beta_H$, is given by

$$\beta_H(T) = \beta \left( \frac{c_{1H}}{c_{2H}} / \frac{Y_{1H}}{Y_2} \right)^{-1} = \beta \left( e_H - T \frac{1 - \lambda_H}{\lambda_H} / Y_1(T) \right)$$

and the aggregate Euler equation is

$$\frac{1}{Y_1(T)} = \beta_H(T) (1 + r) \frac{1}{Y_2}$$

so

$$\frac{\partial Y_1(T)}{\partial T} = - \frac{Y_1(T)}{\beta_H(T)} \frac{\partial \beta_H(T)}{\partial T} \quad (A.4)$$

where

$$\frac{\partial \beta_H(T)}{\partial T} = - \frac{1 - \lambda_H \beta_H(T)}{\epsilon_H \lambda_H Y_1(T)}$$

Thus, if we know how a fiscal policy shock in the detailed economy affect the discount factor in our representative agent formulation, $\beta_H(T)/\partial T$, we can calculate the response of output to $T$ by calculating the response of output to the change in the discount factor in the representative agent formulation.

We now show how we can use the logic in Auclert, Rognlie, and Straub (2018) to express the change in output as a function of the intertemporal MPCs. Let $x = (Y_1, T)$ and note that the solution to the household problem can be expressed as functions $c_{1i}(x), c_{2i}(x)$. Market clearing in the consumption good market requires

$$Y_1(T) = \sum_i \lambda_i c_{1i}(Y_1, T)$$

totally differentiating the expression above we obtain

$$dY_1 = \sum_i \lambda_i \left( \frac{\partial c_{1i}}{\partial Y_1} dY_1 + \frac{\partial c_{1i}}{\partial T} dT + \frac{\partial c_{1i}}{\partial \tau_i} \frac{\partial \tau_i}{\partial T} dT \right)$$

Letting $MPC_{i,(t,j)}$ be agent $i$’s marginal propensity to consume in period $t$ income earned in period $j$, we can write

$$\frac{\partial c_{1i}}{\partial T} = MPC_{i(1,1)} = \begin{cases} \frac{1}{1+\beta} & \text{if } i = H \\ 1 & \text{if } i = L \end{cases}$$

$$\frac{\partial c_{1i}}{\partial \tau_i} = MPC_{i(1,2)} = \begin{cases} \frac{1}{(1+\beta)(1+r)} & \text{if } i = H \\ 0 & \text{if } i = L \end{cases}$$
\[
\frac{\partial c_{1i}}{\partial Y} = e_i \text{MPC}_i(1, 1)
\]

Thus we can combine the expressions above to obtain

\[
\frac{\partial Y_1}{\partial T} \left( 1 - \sum_i \lambda_i e_i \text{MPC}_i(1, 1) \right) = \sum_{t=1}^{2} \sum_i \lambda_i \text{MPC}_i(1, t) dI_{t,i}
\]

where \(dI_{t,i}\) is the direct income change induced by the fiscal policy to agent \(i\) in period \(t\):

\[
dI_{t,i} = \begin{cases} 
1 & \text{if } t = 1 \\
-\frac{1}{\lambda_H} & \text{if } t = 2, i = H \\
0 & \text{if } t = 2, i = L
\end{cases}
\]

Thus

\[
\frac{\partial Y_1}{\partial T} = \frac{\sum_{t=1}^{2} \sum_i \lambda_i \text{MPC}_i(1, t) dI_{t,i}}{(1 - \sum_i \lambda_i e_i \text{MPC}_i(1, 1))} = -\frac{1 - \lambda_H}{\lambda_H} \frac{1}{\lambda_H}
\]

Comparing (A.4) with (A.5) we have:

\[
\frac{\sum_{t=1}^{2} \sum_i \lambda_i \text{MPC}_{i,11} dI_{t,i}}{(1 - \sum_i \lambda_i e_i \text{MPC}_{i,11})} = -\frac{Y_1(T)}{\beta_H(T)} \frac{\partial \beta_H(T)}{\partial T}
\]

Our approach has the advantage that it can be more easily implemented without the need for natural experiments that are necessary for estimating intertemporal MPCs. However, while we can measure \(\beta_i\) from the data, without knowledge of the impulse response functions (e.g. \(\frac{\partial \beta_H(T)}{\partial T}\)), we cannot study how imperfect risk sharing affects the propagation of specific structural shocks (e.g. a fiscal policy shock).

**B Monte Carlo analysis for Krusell and Smith (1998)**

In this appendix we study how well our approximations for estimating the realization of the wedges from panel data and approximating their stochastic processes work in practice by performing a Monte Carlo analysis on data simulated from the Krusell and Smith (1998) economy.

The heterogeneous agents economy. We consider an economy with capital accumulation and flexible prices, \(\kappa = 0\). Households inelastically supply labor, \(\chi = 0\), and their idiosyncratic labor productivity follows a Markov process and can take on two values, \(e \in \{1, 0\}\).
The households decide how much to consume and save, and financial markets are incomplete in that households can only trade claims on the capital stock subject to a debt limit.

The representative firm produces a final good using the technology

\[ Y_t = A_t K_t^\alpha L_t^{1-\alpha}, \]

where \( A_t \) is an aggregate TFP shock, \( K_t \) is the capital stock, and \( L_t \) is total worked hours worked in efficiency units. Given the process for idiosyncratic risk, the latter equals to

\[ L_t = 1 - u_t, \]

where \( u_t \) is the fraction of agents that are currently unemployed.

We let \( A_t \) to take two values, \( A_L < 1 < A_H \). In addition, we allow for the aggregate shock to affect the distribution of idiosyncratic labor productivities: a low \( A_t \) is associated with a higher probability that a household samples \( e = 0 \), which makes idiosyncratic risk countercyclical.

We consider two calibrations. The first is the one in Krusell and Smith (1998). It is well know that with those parameters’ values the model delivers “approximate aggregation”, in the sense that the cyclical behavior of output, consumption and investment in the incomplete market economy closely mirror those in the corresponding economy with complete financial markets—the representative-agent real business cycle model. The second, which we label “high risk calibration”, is identical to the one in Krusell and Smith (1998) with the exception that the probability of being unemployed when \( A_t = A_L \) is 20% instead of 10%. This makes idiosyncratic income risk more countercyclical and, as we will see below, it breaks approximate aggregation. We numerically solve the heterogeneous agent economy using the software provided by Maliar, Maliar, and Valli (2010).

The circled lines in Figure A-1 reports the impulse response functions (IRFs) to a negative TFP shock. Panel (a) reports the IRFs with the Krusell and Smith (1998) calibration: consumption, investment and output fall after the shock, and we know from their analysis that these magnitudes are essentially identical to those of the corresponding representative agent economy. Panel (b) reports the IRFs in the high risk calibration. Relative to the calibration

\[ u_L = \frac{1}{u_H} < u_H, \]

When doing so, we normalize \( A_L \) so that \( A_t(1-u_t) \) is the same across the two calibrations. We compute non-linear IRFs following Koop, Pesaran, and Potter (1996). Starting from the ergodic mean of the model, we compute \( 2 \times M \) simulations for aggregate consumption, investment and output of length \( T \). In the first \( M \) simulations, we restrict TFP at \( t = 1 \) to equal \( A_H \). In the second set of simulations, we restrict TFP at \( t = 1 \) to equal \( A_L \). To obtain the IRFs, we average the first and second sets of simulations across \( M \) and take the difference between the two paths.

\[ A-7 \]
Figure A-1: IRFs to a negative TFP shock

Notes: The circled line reports IRFs to a negative technology shock in the Krusell and Smith (1998) economy. The solid line reports the Monte Carlo average of the IRFs to a negative technology shock in the RA representation, while the dotted line reports the 5th and 95th percentile across the Monte Carlo simulations. Panel (a) reports this experiment under the standard calibration of the Krusell and Smith (1998), while panel (b) reports the same information for the high risk calibration. Consumption, investment and output are reported in percentage changes from their ergodic mean value.

of Krusell and Smith (1998), consumption falls by more and investment falls by less after the shock. This difference is due to the higher incidence of precautionary savings: in the high risk calibration, households have more incentives to save after a negative TFP shock because of the higher probability of being unemployed, and these precautionary motives depress aggregate consumption and increase aggregate investment, as households in this economy can save only by accumulating claims on the capital stock.

The RA representation. We now describe the RA representation of this economy. In what follows, it is convenient to define $\tilde{A}_t = A_t(1 - u_t)^{1-\alpha}$. The exogenous aggregate state for this economy is $z_t = z_t^{RA} = \tilde{A}_t$ and there are no exogenous shocks that are not part of the RA representation, $z_t^{HA} = \emptyset$. The endogenous aggregate state is $X_t = [X_t^{RA}, X_t^{HA}]$ where $X_t^{RA}$ is the level of the capital stock $K_t$ and $X_t^{HA}$ is the joint distribution of asset holdings and individual productivities $\Psi_t$. If one follows the common practice of recording only the mean capital stock as a statistic for the distribution of asset holdings then there is no problem of a missing state variable because $K_t = X_t^{RA}$. 

A-8
Given that labor is inelastically supplied, there is no labor supply condition of households, so \( T_{t+1} = [\hat{\beta}_{1,t+1}, \ldots, \hat{\beta}_{i,t+1}]' \) does not contain \( \omega_t \). Equation (30) then specializes to
\[
T_{t+1} = \Phi(L) \times T_t + A \times \hat{K}_t + B \times \hat{A}_t + C \times \hat{A}_{t+1} + \varepsilon_{t+1}. (A.6)
\]
So, the RA representation is
\[
Y_t = K_{t+1} + C_t
Y_t = \tilde{A}_t K_t^\alpha
1 = \beta \max_i \mathbb{E}_t \left\{ \beta_{i,t+1} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[ \alpha \frac{Y_{t+1} + (1 - \delta) C_{t+1}}{K_{t+1}} \right] \right\}, (A.7)
\]
subject to the stochastic process for \( \tilde{A}_t \) and \( T_t \). This is a representative agent real business cycle model with a time-varying discount factor for the stand-in household, and can be easily solved numerically.

**Monte Carlo analysis.** We now study how well our procedure approximates the IRFs reported in Figure A-1. For that purpose, we proceed as follows. We simulate 500 panel datasets containing households’ level information on consumption, income and assets from the original heterogeneous agent economy. Each panel has 10000 households, and lasts 100 quarters. For each of these panel datasets, and for each \( t \), we partition households in four groups based on their labor income, \( y_{i,t} \), and net worth, \( n_{i,t} \), as described in Section 4.1 and compute \( \beta_{g,t+1} \) using equation (26). We next estimate the stochastic process (A.6), solve for the policy functions of the RA representation, and compute the IRFs to a negative TFP shock.

Let us start by studying how the \( \beta_{g,t+1} \) varies across households. The first column of Table A-1 reports the Monte Carlo average of the sample mean of \( \beta_{g,t+1} \) for each group. We can see that high income households have, on average, a higher \( \beta_{g,t+1} \) than households with lower labor income, especially in the high-risk calibration. This is the result of two forces in the model. First, consumption shares are positively related to idiosyncratic productivity shocks because idiosyncratic risk is not perfectly insured. Second, these productivity shocks are mean-reverting. Thus, the consumption shares of households that are hit by a positive idiosyncratic shock today fall on average between today and tomorrow, explaining why this group has a higher \( \beta_{g,t+1} \) on average.

To verify whether the aggregate Euler equation in (A.7) holds when using the measured
Table A-1: Summary statistics for $\beta_{g,t+1}$

<table>
<thead>
<tr>
<th></th>
<th>Krussel and Smith calibration</th>
<th>High Risk calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean($\hat{\beta}_{g,t+1}$)</td>
<td>Mean($E_{g,t+1}$)</td>
</tr>
<tr>
<td>$y_{L,t}/n_{L,t}$</td>
<td>0.004</td>
<td>-0.060</td>
</tr>
<tr>
<td>$y_{L,t}/n_{H,t}$</td>
<td>0.015</td>
<td>-0.015</td>
</tr>
<tr>
<td>$y_{H,t}/n_{L,t}$</td>
<td>0.015</td>
<td>-0.014</td>
</tr>
<tr>
<td>$y_{H,t}/n_{H,t}$</td>
<td>0.015</td>
<td>-0.014</td>
</tr>
</tbody>
</table>

Notes: For each calibration, the first column reports the Monte Carlo average of the sample mean of $\hat{\beta}_{g,t+1} \equiv (\beta_{g,t+1} - 1) \times 100$, the second column the Monte Carlo average of the sample mean of $EE_{g,t+1}$ defined in equation (A.8), and the third column reports the Monte Carlo average of the $R^2$ of an OLS regression of $\tilde{A}_t$ and $K_t$ on $EE_{g,t+1}$.

wedges, we compute the following statistic

$$EE_{g,t+1} = 100 \times \left( \frac{1}{\beta} - \beta_{g,t+1} \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} R_{k,t+1} \right), \quad (A.8)$$

where $R_{k,t+1}$ is the realized return to capital in period $t+1$. By construction $EE_{g,t+1}$ should be on average equal to zero and orthogonal to the current information set for the households that are on their Euler equation—those achieving the “max” in the Euler equation of the RA representation. In the second column of Table A-1 we report the Monte Carlo average of the sample mean of $EE_{g,t+1}$, while the third column reports the Monte Carlo average of the $R^2$ of an OLS regression of $\tilde{A}_t$ and $K_t$ on $EE_{g,t+1}$. We can see that $EE_{g,t+1}$ associated to high income households are close to zero on average, and they are not predicted by the aggregate state variables of the model. These results show that our approach for measuring the wedge, despite the approximations discussed in Section 4.1, works well in this economy.

After retrieving the time path for the wedges, and for each Monte Carlo replication, we estimate the stochastic process in equation (A.6) with one lag in the autoregressive component. We include in $T_t$ only the $\beta_{g,t}$ for the high income/high net worth group, as the results in Table A-1 shows that the aggregate Euler equation holds when using this wedge.33 Given the estimated parameters, we next solve for the policy function of the equivalent RA representation in (A.7) and compute IRFs to a negative TFP shock. As we repeat this process for every Monte Carlo replication, we obtain a distribution of IRFs.

Figure A-1 reports the results: the solid line represents the average across the Monte Carlo replications while the dotted lines report the 5th and 95th percentile. We can see

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33That is to say, households that have above median income and above median net worth are most of the time unconstrained in this economy, so they achieve the “max” in the Euler equation (A.7). Results are virtually identical when we include in $T_t$ the $\beta_{g,t+1}$ for the four groups.
that in both calibrations the RA representation does remarkably well in retrieving the true underlying IRFs of the heterogeneous agent economy. In the Krussel and Smith calibration, $E_t[\beta_{g,t+1}]$ does not change much in response to the TFP shock. That is, the IRFs in the heterogeneous agent economy are very close to those of a representative agent economy with a time-invariant discount factor. This is an implication of approximate aggregation. In the high risk calibration, instead, $E_t[\beta_{g,t+1}]$ increases substantially when TFP falls. That is, the RA representation captures the countercyclical precautionary motives that arise in the original heterogeneous agent economy through an increase in the discount factor of the stand-in household.

C Data

In this appendix we give more details about sample selection and variables definition. We also present some summary statistics of the raw data and show that our sample both aggregates reasonably and is consistent with recent work on consumption inequality.

C.1 Definition of variables and sample selection in the CEX

Consumption expenditures. Our measure of consumption expenditure is close to the NIPA definition of nondurable and services expenditures. It is constructed by aggregating up the following expenditure sub-categories: food, tobacco, domestic services, adult and child care, utilities, transportation, pet expenses, apparel, education, work-related and training, healthcare, insurance, furniture rental and small textiles, housing related expenditures excluding rent.

Total hours worked. We compute total hours worked for the head of household by multiplying the number of weeks worked full or part time over the last year ($INCWEEKI$) multiplied by the numbers of hours usually worked per week ($INC_HRSI$). We obtain the same indicator for the spouse and add the two.

Labor income. We compute labor income as the sum of total household (CU) income from earnings before taxes ($FSALARYI$), plus the total income received from farm ($FFRMINCI$) and nonfarm business ($FNONFRMI$).

Liquid assets. It includes the total amount the households held in savings accounts in financial institutions ($SAVACCTI$), checking and brokerage accounts ($CKBKACTI$). In the CEX, these amounts are only reported in the last interview. Thus they represent end of period
values for the household. In order to define beginning of period values for these assets, we use the following variables (COMPSAVI and COMPCKGI), which report the total change in savings and checking accounts over the previous year, respectively. Then beginning of period values are defined as end of period values minus the change in value.

**Illiquid Assets.** It includes the value of owned automobiles (NETPURI), residential housing (PROPVALI), U.S. savings bonds (USBNDI), the value of all securities directly held by the household (include stocks, mutual funds and non U.S. savings bonds) (SECESTI), and money owned to the household by individuals outside of the household (MONEYOWDI). The value of U.S. savings bonds and total securities are only reported in household interview. In order to define beginning of period values for these assets, we use the following variables (COMPBNDI and COMPSECI), which report the total change in U.S. savings bonds and all securities over the previous year, respectively. Then beginning of period values are defined as end of period values minus the change in value.

**Total assets.** It is the value of liquid assets plus illiquid assets each household owns.

**Liabilities.** It is the current value of the household’s home mortgage (QBLNCM3I) plus the outstanding principal balance on auto debt. (QBALNM3I).

**Net worth.** Net Worth is total assets minus liabilities.

The baseline sample includes all households where the head of the household is between the ages of 22 and 64. We only use households who participate in all four interviews in the CEX. We restrict the sample to those which the CEX labels as "complete income reporters," corresponding to households with at least one non-zero response to any of the income and benefits questions. We use the assigned "replicate" or sample weights, designed to map the CEX into the national population in all calculations. We use the CPI-U to express all monetary variables in constant 2000 dollars. To eliminate outliers and mitigate any impact of time-varying top-coding, we drop observations in the top and bottom one percent of the consumption, hours, labor income, wage per hour, total assets, total liabilities and net-worth distribution.

Taking 2006 as the year of reference, we have 5180 households that report full consumption information in all four interviews. We next keep households whose head is in the age bracket 22-64, leaving us with 3890 households that reported income and consumption in 2006. Within this group, we keep households that are considered “full income responders” (3163), and drop any household that observed a change in family size between the first and the last interview (2736). We then drop observations on consumption, labor income,
C.2 Summary statistics

Table A-2 reports selected households’ characteristics for 2006. In the CEX, the average age for the head of household was 44 years, and roughly 34% of the households’ head held a college degree. The average size of the household was 2.7. On average, households spent roughly 10000 dollars per person in non-durables and services, and the average income per person was 26000 dollars. Households worked 1300 hours per year per person on average, earning an average wage of 19.80 dollars per hour. The mean net worth for the household was 142000 dollars, with 14000 dollars in liquid assets. As a comparison with previous papers, the average characteristics of the household in our sample are very close to those reported in Heathcote and Perri (2018), see Table 1 in their paper.

C.3 Aggregation

In this section we examine whether the dynamics of aggregate consumption, income, and total hours per capita in our cross sectional data capture the broad contours of national
income and product accounts (NIPA) aggregates. The results are shown in figure A-2. Each series is normalized to 1 in 2004.

The top left panel of figure A-2 shows the dynamics of average per capita expenditures in the CEX and the equivalent measure in the NIPA. The top left panel shows average per capita disposable income in the CEX and NIPA. The bottom panel shows average total hours worker per capita in the CEX as well as its aggregate counterpart obtained from the BLS. While the fit is not perfect, it is clear that the dataset captures the broad contour of each aggregate series during the Great Recession.

C.4 Trends in inequality

A large literature has documented that consumption inequality has increased in the U.S. since 1980 (Blundell, Pistaferri, and Preston, 2008; Krueger and Perri, 2006; Aguiar and Bils, 2015; Attanasio and Pistaferri, 2016). Consistent with this literature, we find that the variance

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34 These figures are constructed before any sample selection.
35 Our figure A-2 is very similar to the relevant panels in Figure 13 of Heathcote and Perri (2018) giving us further confidence.
on log consumption has increased significantly in the CEX. These results are displayed in figure A-3. There is clear visual evidence that consumption volatility has increased. Moreover, the levels of consumption inequality that we find are very similar to previous work in the literature. In particular, we find that the variance of log consumption has increased from 0.23 to 0.28 in the CEX over the period 1985 to 2005, which is almost the exact same increase in both levels and changes that Heathcote, Perri, and Violante (2010) find over the same time period (see figure 1 in the recent survey by Attanasio and Pistaferri (2016) for more details). Overall, this suggests that our sample selection procedure is reasonable.

C.5 $\beta$ measurement robustness

Figure 2 shows the results of our baseline measurement of the (max) $\beta_{g,t}$ where we grouped households according to whether their income at date $t-1$ is above or below median income and, within each of these two groups, whether the level of their net worth is above or below the group median. Figure A-4 reports results when we use partition households using the following different $t-1$ state variables: two income and two liquid asset groups (red dashed line), two income and two total asset groups (blue dotted line), and four income groups (green dash-dotted line). Our baseline group is shown in the black dotted line. The main takeaway from this exercise is that all four time series look very similar with each showing a
similarly large increase during the Great Recession. This suggests that our results are robust to using other natural partitions of the CEX data.

Figure A-5 shows a different robustness check on our measurement of $\beta_{g,t}$. Here we report results for a baseline partition where we group households according to whether their income and net worth at date $t-1$ in the bi-annual Panel Survey of Income Dynamics (PSID) over the period 1999-2015. Concretely, at each $t-1$, we end up with four different groups of households of approximately equal size: low income/low net worth, low income/high net worth, high income/low net worth and high income/high net worth. For each group $i$, we use equation (26) to construct $\beta_{g,t}$ in the PSID. Comparing Figure A-5 with Figure 2, we see there are many broad similarities. In particular, the high income groups have higher discount factor wedges relative to low income households in most years (particularly so for the high income/high net worth group). That is, we find similar relative rankings of groups in both the CEX and the PSID. This suggests that our measurement approach is capturing relevant variation in the data and not just measurement error.
Figure A-5: Measured $\beta$ in the PSID

Notes: Each panel shows an estimate of $\beta_{it}$ in the PSID for four groups of roughly equal size: low income/low net worth, low income/high net worth, high income/low net worth and high income/high net worth (red dashed line) along with the year-by-year value of the maximum across the four groups (solid black line) over the periods 1999-2015. Over this period the PSID is bi-annual.
Let the state vector be $\mathbf{S}_t = [i_{t-1}, \hat{\theta}_t, \hat{A}_t, \epsilon_{m,t}, \hat{\beta}_t, \hat{\omega}_t]$. The equilibrium conditions of the model can be summarized by the following equations

\[
Y(S_t) = C(S_t) + \frac{\kappa}{2} \left( \frac{\pi(S_t) - \pi^*}{1 + \pi^*} \right)^2 \tag{A.9}
\]

\[
Y^{\text{pot}}(S_t) = \left[ \frac{\exp(\tilde{A}_t)^{1+\psi}}{\exp(\tilde{\omega}_t) \mu \chi} \right]^{1/\psi} \tag{A.10}
\]

\[
1 + i(S_t) = \max \left\{ (1 + i_{t-1})^{\psi^*} \left( 1 + \tilde{i} \left( \frac{1 + \pi(S_t)}{1 + \pi^*} \right)^{\psi^*} \left( \frac{Y(S_t)}{Y^{\text{pot}}(S_t)} \right)^{\psi^*} \right)^{(1-\rho^*)}, 1 \right\} \tag{A.11}
\]

\[
1 + \frac{\pi(S_t) - \pi^*}{1 + \pi^*} = \frac{1}{\kappa(\mu - 1)} Y(S_t) \left( \mu Y(S_t)^{\psi} C(S_t)^{\sigma} \exp(\tilde{\omega}_t) \right) - 1
\]

\[
+ \beta E_t \left[ \exp(\tilde{\theta}_{t+1} + \beta_{t+1}) \left( \frac{C(S_{t+1})}{C(S_t)} \right)^{-\sigma} \pi(S_{t+1}) - \pi^* \frac{1 + \pi(S_{t+1})}{1 + \pi^*} \right]. \tag{A.13}
\]

Given policy functions for $C(S_t)$ and $\pi(S_t)$, we can use equations (A.9)-(A.11) to solve for $Y(S_t)$ and $i(S_t)$. Thus, the numerical solution of the model can be equivalently expressed as approximating $C(S_t)$ and $\pi(S_t)$.

Due to the max operator in equation (A.11), $C(S_t)$ and $\pi(S_t)$ may have kinks in a region of $S_t$ where the zero lower bound constraint starts binding, a feature that makes it challenging to approximate these functions with smooth polynomials. We approach this feature following Gust et al. (2017). Specifically, we approximate these variables using a piece-wise smooth function,

\[
x(S_t) = \mathbb{1}(1 + \tilde{i}(S_t) > 1) \gamma_x^{\text{no zlb}} T(S_t) + \mathbb{1}(1 + \tilde{i}(S_t) \leq 1) \gamma_x^{\text{zlb}} T(S_t), \tag{A.14}
\]

where $x = \{C, \pi\}$, $1 + \tilde{i}(S_t)$ is the “notional” interest rate at $S_t$ (the first term inside the max operator of equation (A.11)), $T(S_t)$ is a vector collecting Chebyshev’s polynomials evaluated at $S_t$ and $\{\gamma_x^{\text{no zlb}}, \gamma_x^{\text{zlb}}\}$ a set of coefficients.

The numerical solution of the model consists in choosing $\Gamma = \{\gamma_x^{\text{no zlb}}, \gamma_x^{\text{zlb}}\}_{x=C,\pi}$ so that equations (A.12) and (A.13) are satisfied for a set of collocation points $\tilde{S}_i \in S$. The choice of collocation points and the associated Chebyshev’s polynomials follows the method of Smolyak. Conditional expectations in equations (A.12) and (A.13) are evaluated using Gauss-Hermite quadrature.
The algorithm for the numerical solution of the model is as follows:

**Step 0.A: Defining the grid and the polynomials.** Set upper and lower bounds on the state variables \( \tilde{\mathbf{S}} = [\tilde{i}, \tilde{\theta}, \tilde{A}, \tilde{\varepsilon}_m, \tilde{\beta}, \tilde{\omega}] \). Given these bounds, construct a Smolyak grid and the associated Chebyshev’s polynomials.

**Step 1: Equilibrium conditions at the candidate solution.** Start with a guess for the model’s policy functions \( \Gamma^c \). For each \( \tilde{\mathbf{S}} \), compute \( C^{\text{no zlb}}(\tilde{\mathbf{S}}) \), \( C^{\text{zlb}}(\tilde{\mathbf{S}}) \), \( \pi^{\text{no zlb}}(\tilde{\mathbf{S}}) \), and \( \pi^{\text{zlb}}(\tilde{\mathbf{S}}) \) using the coefficients in \( \Gamma^c \). Evaluate equation (A.9) using \( C^{\text{no zlb}}(\tilde{\mathbf{S}}) \) and \( \pi^{\text{no zlb}}(\tilde{\mathbf{S}}) \) to obtain \( Y^{\text{no zlb}}(\tilde{\mathbf{S}}) \), and similarly obtain a value for \( Y^{\text{zlb}}(\tilde{\mathbf{S}}) \). Use equation (A.10) and (A.11) along with \( Y^{\text{no zlb}}(\tilde{\mathbf{S}}) \) and \( \pi^{\text{no zlb}}(\tilde{\mathbf{S}}) \) to obtain the notional interest rate \( 1 + \tilde{i}(\tilde{\mathbf{S}}) \). Compute the actual interest rate \( 1 + i(\tilde{\mathbf{S}}) = \max\{1 + \tilde{i}(\tilde{\mathbf{S}}), 1\} \).

**Step 3: Evaluate residual equations.** For each \( \tilde{\mathbf{S}} \), compute the residual equations

\[
\mathcal{R}^1(\tilde{\mathbf{S}}) = \left[ \frac{1}{1 + i(\tilde{\mathbf{S}})} \right] - \beta \mathbb{E} \left[ \exp\{\theta' + \beta'\} \left( \frac{C(S')}{C^{\text{no zlb}}(\tilde{\mathbf{S}})} \right)^{-\sigma} \frac{1}{1 + \pi(S')} \right]
\]

\[
\mathcal{R}^2(\tilde{\mathbf{S}}) = 1 - \beta \mathbb{E} \left[ \exp\{\theta' + \beta'\} \left( \frac{C(S')}{C^{\text{zlb}}(\tilde{\mathbf{S}})} \right)^{-\sigma} \frac{1}{1 + \pi(S')} \right].
\]

Similarly, compute \( \mathcal{R}^3(\tilde{\mathbf{S}}) \) and \( \mathcal{R}^4(\tilde{\mathbf{S}}) \) using equation (A.13).

**Step 4: Iteration.** Let \( \mathcal{R}(\Gamma^c) \) the vector collecting all the computed residuals at the collocation point, and let \( r \) be its Euclidean norm. If \( r \leq 10^{-10} \), stop the algorithm. If not, update the guess and repeat Step 1-4. \( \square \)

The specifics for the algorithm are as follows. The bounds on \( [\tilde{\theta}, \tilde{A}, \tilde{\varepsilon}_m, \tilde{\beta}, \tilde{\omega}] \) are +/- 3 standard deviations from their mean. The bounds on \( \tilde{i} \) is set to \([0, 0.20]\), wide enough to span the ergodic distribution of nominal interest rates. We consider a second-order Smolyak grid, and use 243 points for Gauss-Hermite quadrature (three points for each shock and tensor multiplications). Finally, we use a Newton algorithm to find the zeros of \( \mathcal{R}(\Gamma^c) \) at the collocation points.

### E Quantitative analysis

In this Appendix we present additional details regarding the quantitative experiments of Section 5. We start with the estimation of the model and then discuss in details the counterfactual of Section 5.4.
Table A-3: Prior and posterior distribution: structural parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior</th>
<th>St. dev.</th>
<th>Mean</th>
<th>90% Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$4 \times \kappa$</td>
<td>Gamma</td>
<td>85.00</td>
<td>15.00</td>
<td>67.13</td>
<td>[55.66, 77.52]</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
<td>0.34</td>
<td>[0.10, 0.55]</td>
</tr>
<tr>
<td>$\gamma_\pi$</td>
<td>Normal</td>
<td>1.50</td>
<td>2.00</td>
<td>2.52</td>
<td>[1.31, 3.78]</td>
</tr>
<tr>
<td>$\gamma_y$</td>
<td>Normal</td>
<td>1.00</td>
<td>2.00</td>
<td>-0.17</td>
<td>[-0.29, -0.06]</td>
</tr>
<tr>
<td>$\rho_\theta$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.28</td>
<td>0.84</td>
<td>[0.72, 0.95]</td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.28</td>
<td>0.79</td>
<td>[0.68, 0.91]</td>
</tr>
<tr>
<td>$100 \times \sigma_\theta$</td>
<td>InvGamma</td>
<td>1.00</td>
<td>5.00</td>
<td>0.91</td>
<td>[0.44, 1.33]</td>
</tr>
<tr>
<td>$100 \times \sigma_A$</td>
<td>InvGamma</td>
<td>1.00</td>
<td>5.00</td>
<td>2.43</td>
<td>[1.67, 3.11]</td>
</tr>
<tr>
<td>$100 \times \sigma_m$</td>
<td>InvGamma</td>
<td>1.00</td>
<td>5.00</td>
<td>1.40</td>
<td>[0.81, 2.03]</td>
</tr>
</tbody>
</table>

Notes: The posterior statistics reports the mean, fifth and ninety-fifth percentile of the posterior distribution estimated by pooling 2 Markov chains with 100,000 draws each (including a 100,000 draw burn-in period for each chain).

### E.1 Model estimation

Draws from the posterior distribution of the model parameters are generated using the random walk Metropolis Hastings described in An and Schorfheide (2007). The proposal distribution is a multivariate normal, with variance-covariance matrix given by $c\Sigma$, where $\Sigma$ is the negative of the inverse hessian of the log-posterior evaluated at the posterior mode and $c$ is a constant that we set to obtain roughly a 30% acceptance rate in Markov chain. We generate 2 Markov chains of 200,000 each, and discard the first 100,000 draws in each chain. The statistics of the posterior distribution of model parameters reported in Table A-3 and Table A-4 are computed by combining the last 100,000 draws for each chain.

### E.2 Counterfactuals

We now detail the counterfactual experiment of Section 5. We first explain how we use the particle filter to obtain an estimate of the structural shocks. Next, we discuss how we generate the decomposition of Figure 5.

The RA representation implies the following law of motion for the model’s variables

\[
Y_t = g(S_t) + \eta_t \\
S_t = f(S_{t-1}, \varepsilon_t).
\]

The first set of equations describes the evolution of the observables $Y_t$, with $\eta_t$ being a vector of iid Gaussian errors with a diagonal variance-covariance matrix equal to $H$. The
Table A-4: Prior and posterior distribution: stochastic process of the wedges

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior</th>
<th>St. dev.</th>
<th>Mean</th>
<th>90% Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Phi_{\beta,\beta}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
<td>0.34</td>
<td>[0.19, 0.50]</td>
</tr>
<tr>
<td>$\Phi_{\omega,\beta}$</td>
<td>Beta</td>
<td>0.50</td>
<td>0.25</td>
<td>0.12</td>
<td>[-0.09, 0.33]</td>
</tr>
<tr>
<td>$\Phi_{\omega,\omega}$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.50</td>
<td>0.80</td>
<td>[0.66, 0.93]</td>
</tr>
<tr>
<td>$\Phi_{\omega,\epsilon_m}$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.50</td>
<td>0.52</td>
<td>[0.16, 0.95]</td>
</tr>
<tr>
<td>$\Phi_{\epsilon_m,\epsilon_m}$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.50</td>
<td>0.29</td>
<td>[0.19, 0.39]</td>
</tr>
<tr>
<td>$\Phi_{\epsilon_m,\omega}$</td>
<td>Normal</td>
<td>0.00</td>
<td>0.50</td>
<td>0.31</td>
<td>[0.15, 0.47]</td>
</tr>
</tbody>
</table>

Notes: This table reports estimates for the parameters in (30). The parameter $A_{x,y}$ stands for the loading of an element $y$ of $X$ on the variable $x$ in $T_{t+1}$. These parameters define the matrix $A$ in (30). The parameters $B_{x,y}$ and $C_{x,y}$ are defined in a similar manner. The posterior statistics are constructed as explained in the note to Table A-3.

The second equation describes the evolution of the state variables $S_t$. The vector $\epsilon_t$ collects the innovations to the structural shocks $\hat{\theta}_t$, $\hat{A}_t$, and $\epsilon_{m,t}$ and the preference wedges $\hat{\beta}_t$, $\hat{\omega}_t$ and $\hat{\omega}_{cm,t}$. The functions $g(\cdot)$ and $f(\cdot)$ are generated using the numerical algorithm described.
Fix the vector of model parameters $\varphi$, and denote by $Y^t = [Y_1, \ldots, Y_t]$ the vector collecting data and by $p(S_t|Y^t)$ the conditional distribution of the state vector given observations up to period $t$. Although the conditional density of $Y_t$ given $S_t$ is known and Gaussian, there is no analytical expression for the density $p(S_t|Y^t)$. We use the particle filter to approximate this density for each $t$. The approximation is done via a set of pairs $\{S^i_t, \tilde{w}^i_t\}_{i=1}^N$, in the sense that

$$\frac{1}{N} \sum_{i=1}^N f(S^i_t) \tilde{w}^i_t \overset{a.s.}{\to} \mathbb{E}[f(S_t)|Y^t].$$

We refer to $S^i_t$ as a particle and to $\tilde{w}^i_t$ as its weight. The algorithm used to approximate $\{p(S_t|Y^t)\}_t$ builds on Kitagawa (1996), and it goes as follows:

**Step 0: Initialization.** Set $t = 1$. Initialize $\{S^i_0, \tilde{w}^i_0\}_{i=1}^N$ and set $\tilde{w}^i_0 = 1 \forall i$.

**Step 1: Prediction.** For each $i = 1, \ldots, N$, obtain a realization for the state vector $S^i_{t|t-1}$ given $S^i_{t-1}$ by simulating the model forward.

**Step 2: Filtering.** Assign to each particle $S^i_{t|t-1}$ the weight

$$\tilde{w}^i_t = p(Y_t|S^i_{t|t-1}) \tilde{w}^i_{t-1}.$$

**Step 3: Resampling.** Rescale the weights $\{w^i_t\}$ so that they add up to one, and denote these rescaled values by $\{\tilde{w}^i_t\}$. Sample $N$ values for the state vector with replacement from $\{S^i_{t|t-1}, \tilde{w}^i_t\}_{i=1}^N$, and denote these draws by $\{S^i_t\}$. Set $\tilde{w}^i_t = 1 \forall i$. If $t < T$, set $t = t + 1$ and go to Step 1. If not, stop. □

In our exercise, the measurement equation includes nominal interest rates, linearly detrended real GDP, inflation, $\hat{\beta}_t$, $\hat{\omega}_t$, and $\hat{\omega}_{cm}$. The variance on the measurement errors on the first three variables is set to 0.5% of their unconditional variance, while we set the variance of the measurement errors on the wedges to 10% of their unconditional variance. We set $N$ to 1,000,000. Given the vector of model parameters $\varphi$, we solve the model using the algorithm in Appendix D, apply the particle filter to our data, and obtain an estimate for the latent states. We repeat this procedure for 100 approximately iid draws from the posterior distribution of the model parameters. Figure 4 reports the posterior mean and 90% credible sets for $Y_t$ and $S_t$.

In order to generate the counterfactual of Figure 5, we first solve the model setting $\hat{\beta}_t = 0$, obtaining the policy function $g^{no\hat{\beta}}(.)$ and $f^{no\hat{\beta}}(.)$. We then compute the counterfactual value of a variable $y_t$ as

$$y^\text{counterfactual}_t = \sum_{i=1}^N g^{no\hat{\beta}}(S^i_t) \tilde{w}^i_t.$$
where \( S^i_t = [\hat{\theta}^i_t, \hat{A}^i_t, \hat{\epsilon}^i_m, \hat{\omega}^i_t \times (\hat{\omega}^c_{t \times} / \hat{\omega}_t)] \). We repeat this procedure the 100 draws from the posterior distribution of model parameters in order to construct credible sets.